Discussion Questions Sections 2.1

Most of the items at the top level of the outline refer to specific theorems or definitions. The intent is that you read the statement of the theorem or definition in the text and then answer the discussion questions, before continuing with your reading. Before you start to read, it might help you to mark in the text the points at which you need to stop and answer questions.

1. Equivalence relations and classes should have been covered in Foundations of Mathematics.
    a. What is the definition of an equivalence relation? Of equivalence classes?
    b. What are the important properties of equivalence classes?

2. 1st Definition in section 2.1. Answer these questions before continuing to read on page 25.
    a. Give an example with specific numbers for *a*, *b*, and *n* where *a* IS congruent to *b* modulo *n*, and an example where *a* IS NOT congruent to *b* modulo *n*.
    b. What integers are equivalent to 0 mod *n*?
    c. What integers are equivalent to 3 mod 5?
    d. For what *n* are 55 and 79 equivalent mod *n*?
    e. Congruence modulo *n* is an equivalence relation. What must be proved to show that is true?

3. Theorem 2.1. This establishes that congruence mod *n* is an equivalence relation.
    a. Compare the statement of the theorem with what you answered to 2e.
    b. For each part of the theorem, describe how a proof should be expected to be organized. For example, in part (1), using the preceding definition, we can prove $a \equiv a$ mod n, by showing that $n \mid a - a$. How might the other two parts be proved?
    c. Try to prove the three parts of the theorem without reading the proof in the book.

4. Theorem 2.2
    a. What is the significance of the last four lines on page 25? Can you think of a reason why it would be useful to prove that the "key fact" discussed there is true for congruence? (It is ok if the answer is no: but it is good to have the question in the back of your mind as you continue reading. At some point you should be able to say why theorem 2.2 is worth knowing.)
    b. Make up specific values for *a*, *b*, *c*, *d*, and *n* that satisfy the hypotheses of the theorem. Check whether the conclusions of the theorem hold for your example.
    c. Change *n* in your example to a different number, but keep the other numbers unchanged. Are the hypotheses still satisfied? What about the conclusions?
    d. Again, try to first outline each part of the proof, and then fill in the outline to develop your own proof.

5. Definition on page 28.
    a. The term *class* here has a specific meaning. But you (should) have seen that word in other mathematical contexts. Is there a connection? If so, what properties do you expect congruence classes to have?
    b. What is [7] modulo 5? What is [3] modulo 17? Can you find a graphical or geometric interpretation that applies to congruence classes in general?

6. Theorem 2.3
    a. Make up a particular *n*. Pick 4 or 5 integers at random, and find the congruence class for each one. Verify that the conclusion of the theorem holds for all of your selected integers.
    b. Relate this theorem, if possible, to your graphical or visual interpretation from 5b.
    c. Relate it if possible to your knowledge of equivalence relations. What is the significance of the two sentences between the statement and the proof of this theorem?
    d. Compose an outline that you expect a proof of this theorem to follow (hint: it is an if and only if theorem).
    e. Note that the statement $[a] = [b]$ is an equality of two *sets*. What is the usual procedure for proving such an equality? Incorporate this idea into your outline of a proof.
    f. Does the proof in the text conform to your expected outline? Which part of your outline goes with the first part of the book's proof? The second part?
    g. Verify the correctness of all the steps in the first part of the proof. Be sure you know what definition is referred to in the statement "*…by definition…*", and what *by transitivity* and *by symmetry* mean, and why they apply.

7. Be prepared to state Corollary 2.5, and outline the proof. Likewise the final definition of the section (on page 28).

8. We are accustomed to the fact that rational numbers have many equivalent expressions as fractions: 2/4 = 3/6 = 23/46, etc. Using this idea, develop an analogy between the set $\mathbb{Q}$ of rationals and for example $\mathbb{Z}_{11}$.