

## Extra Problems Section 4.2

1. Solve the initial value problem

$$y'' + 4y = \cos \alpha t; \quad y(0) = y'(0) = 0,$$

where  $\alpha$  is a real constant.

[Solution:  $y(t) = \frac{1}{\alpha^2 - 4}(\cos(2t) - \cos(\alpha t)).$ ]

2. Continuing the prior problem, suppose that  $\alpha = 2 + \varepsilon$ . Use the trigonometric identity  $\cos(A) - \cos(B) = 2 \sin\left(\frac{B-A}{2}\right) \sin\left(\frac{B+A}{2}\right)$  to show that the solution to the initial value problem in 1 can be expressed as

$$y(t) = \frac{2}{\varepsilon(4 + \varepsilon)} \left( \sin\left(\frac{\varepsilon}{2}t\right) \sin\left(\left(2 + \frac{\varepsilon}{2}\right)t\right) \right).$$

3. Use the result of 2 to analyze the graph of  $y(t)$  when  $\varepsilon = .2$ . What is the numerical value of the coefficient  $\frac{2}{\varepsilon(4 + \varepsilon)}$ ? What is the period of the first sine factor? The second sine factor? Using these results, describe the graph.
4. Repeat problem 3 using first  $\varepsilon = .02$  then  $\varepsilon = .002$ . What do these results suggest about the behavior of the system as  $\alpha \rightarrow 2$ ?
5. How do parts 3 and 4 relate to the concept of resonance?