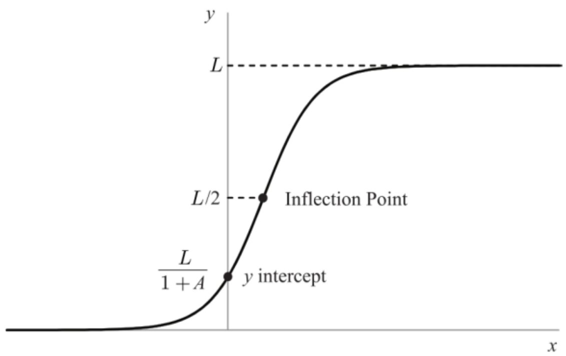
Extra Problems Section 4.4

These problems concern the family of logistic functions $y = \frac{L}{1+Ae^{-kt}}$ where L, A, and k can be any positive constants. It is known that this function has a y intercept at L / (1+A), an inflection point where $y = L/2$, and horizontal asymptotes at $y = 0$ (for $x \to -\infty$) and $y = L$ (for $x \to \infty$). These features are illustrated in the figure below.



4.4.x1. Logistic functions arise as solutions to differential equations. For example, in the handout on Euler's Method, we saw the differential equation $y' = .01(25 - y)y$. The solutions to this differential equation are given by $y = \frac{25}{1+Ae^{-.25x}}$.

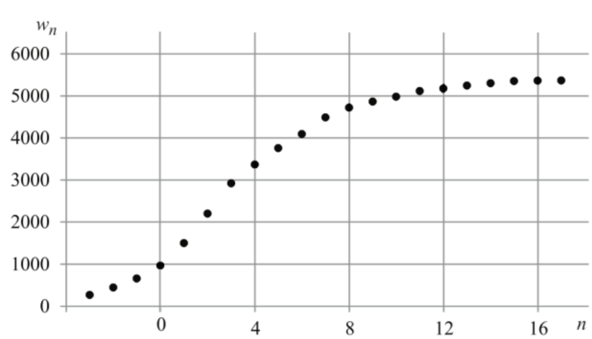
a. Find the solution curve given the condition that $y = 1.2$ when $x = 0$.
b. If we know that $A = 4$, find the inflection point of the curve.
c. Again assuming $A = 4$, find the point at which $y = 24$.

4.4.x2 A logistic curve has an inflection point at (−10,480) and a y intercept of 400. Find the equation of the curve.

4.4.x3 A biologist interested in modeling the growth of various kinds of plants collected data by weighing one particular pumpkin daily for about three weeks. The data are and a graph appear below, with the fourth data point arbitrarily designated to be day 0. Find a logistic curve that approximates the data. [Hints: Estimate the value of L by looking at the data and graph. Use the entry in the table for day 0 as the y intercept, and find A. Estimate the location of the inflection point (assuming $y = L/2$ there), and use that to find k.]

Days	Grams
−3	267
−2	443
−1	658
0	961
1	1498
2	2200
3	2920
4	3366
5	3758
6	4092
7	4488

Days	Grams
8	4720
9	4864
10	4980
11	5114
12	5176
13	5242
14	5298
15	5352
16	5360
17	5366



Answers

4.4.x1.

a. $A = \frac{25}{1.2} - 1 = \frac{119}{6} = 19.8333\ldots$

b. (4 ln 4, 12.5)

c. $x = 4 \ln 96$

4.4.x2. $L = 2\cdot 480 = 960$. $A = \frac{960}{400} - 1 = 1.4$. $k = \frac{\ln 1.4}{-10} = -.1 \ln 1.4$.

4.4.x3. We estimate L as 5370, and find $A = \frac{5370}{961} - 1 \approx 4.588$. We also estimate the x coordinate of the inflection point as 2.5 using the graph. That leads to $k = \frac{\ln 4.588}{2.5} \approx .6094$. The resulting logistic curve is graphed against the data points in the first figure below. After some trial and error variation of the parameters, an improved fit can be found with L, A, and k given by 5370, 5, and .5, respectively. This is shown in the second graph below.