Note: This sample exam was compiled from several different calculus exams, and is too long for an actual in-class exam. The real exam will be shorter. Also, this sample exam is NOT a comprehensive review. The real exam will include questions on topics not included here, and vice versa.

Name ____________________________

Calculus 1
Sample Exam 2

Instructions: This test has 7 problems. Each problem is worth the number of points indicated. Except where stated otherwise, for full credit on each problem you must show work or explain your reasoning. If you use your calculator, (except for simple computations) tell me what you did and what the result was. In short, I must be able to understand how you reached your answer.

1. **(45 points)** Finding Derivatives. There are 6 parts to this problem, on this page and the next two. For each part find the derivative of the given function. You may use any rules or techniques we have studied. For each step either show your work or state what you did.

a. $f(x) = 4x^2 - 4/x^2$

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b. $f(x) = \dfrac{e^x + 4}{e^x - 4}$

Problem 1, continued.

c. $f(x) = \cos x \sin x$

d. $f(x) = \ln(5x^3 + 4x)$

Problem 1, continued.

e. $f(x) = \sqrt{1+(e^x+\tan x)^3}$

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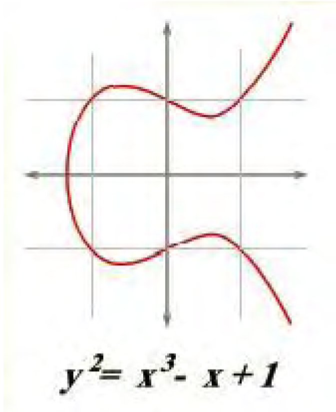
f. Find y' at the point $(1,2)$ on the curve with equation $x^2 + 5xy + 6y^2 = 35$. [Hint: use implicit differentiation.]

2. **(7 points)** The inverse tangent function is defined by the statement: $y = \tan^{-1}(x)$ if and only if $x = \tan y$. Use this fact, and implicit differentiation, to show that $(\tan^{-1}(x))' = 1/(x^2 + 1)$. (Hint: $\sec^2 A = 1 + \tan^2 A$ for any A.)

3. **(10 points)** The graph of the equation $y^2 = x^3 - x + 1$ is shown at right below. Find an equation for the tangent line of this curve at the point $(-1, -1)$.



4. **(10 points)** In a science fair project gone horribly wrong, a gooey blob of slime comes to life and starts consuming everything it touches: students, teachers, parents, annoying younger siblings, etc. A math student figures out that the blob is growing according to the equation $v' = 4.8v$, where $v(t)$ is the volume of the blob in cubic feet at time t, and where t is in units of hours, starting with $t = 0$ at noon on 10/3/2010. If the blob was 30 cubic feet at $t = 0$, how big was it by noon the next day? If the blob cannot be stopped, and keeps growing in the same way, how long will it take for the blob to grow to the size of the entire earth (roughly a sphere with a radius of 20.9 million feet and therefore a volume of $(4\pi/3)(20900000^3)$ cubic feet).

5. True False

a. In developing a differential equation for an evolving system, simplifying assumptions with known errors should generally be avoided.

b. Differential equation methods have proven to be highly effective in predicting future evolution of physical systems in nature, such as motions of planets and satellites, chemical reactions, and the weather.

c. When you solve a normal algebraic equation, the goal is to identify an unknown number, but when you solve a differential equation, the goal is to identify an unknown *function*.

d. Differential equation models can be effective tools for understanding systems in nature, such as chemical reactions or biological processes, but they are not applicable to social science phenomena such as economics, communications, or politics.

e. After the determination of tangent lines and areas, differential equations are the third most common use of calculus in applications.

6. **(10 points)** Differential equation. I am trying to determine an unknown function $y = f(x)$. I know that when $x = 0$, $y = 4$. I also know that the derivative of the unknown function obeys the equation $y' = 3y + 2$. That is, I know that for any x, $f'(x) = 3f(x) + 2$. Find an equation for the unknown function. [Hint: try to rearrange the equation for $f'(x)$ into something of the form $A/B = C$ where C is constant, and where A is the derivative of B.]

7. Let $f(x) = 2 - 0.3\, x^{2/3}\,(x^2 - 9)$ for x in [-2, 1]. Then $f'(x) = -0.8\frac{x^2 - \frac{9}{4}}{\sqrt[3]{x}}$.

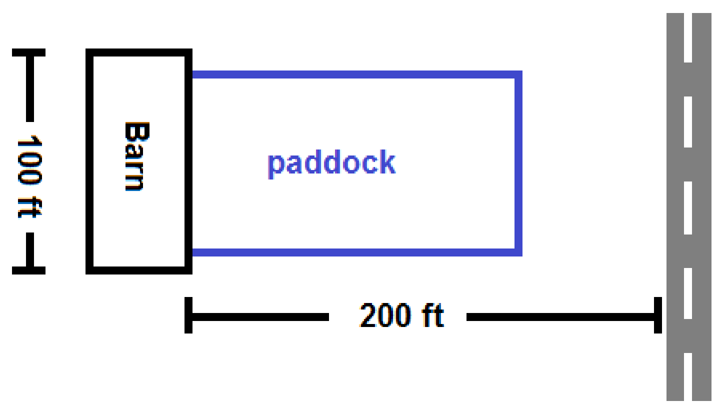
   a. Find all the critical points for f in the given interval. Show work and/or explain your method(s).

   b. Find all local minima and maxima for f on the given interval. Justify your answers.

   c. Find the global maximum (if one exists) and the global minimum (if one exists) for f on the given interval. Explain how you can be sure your method produces the correct answers.

8. Find the global max and min, if any, of $x + 1/x$ for positive x.

9. Fence Problem. A farmer wants to enclose a paddock between his barn and a road, as shown in the diagram. The barn is 100 feet long, and the road is 200 feet from the barn. The farmer has 240 feet of livestock wire mesh available, to enclose three sides of the paddock. No fence will be needed along the side of the barn. Assuming that all of the fencing will be used, what dimensions will provide a paddock with the greatest area?



[For partial credit, translate this into a calc1 max/min problem without solving the problem. That is, find a function of one variable whose global max (or min) on some interval gives the answer to the Fence Problem. Include in your answer a description of what quantity the variable of your function represents, what the interval is, whether you must find the max or min, and how that can be used to determine the answer to the Fence Problem. Also show work or explain how you obtained the equation of your function.]