Applied Calculus 1
Spring 2000
Exam 3 Name ______________________________

Instructions: This test has 7 problems, some with several parts, with a total of 100 points. For full credit on non-essay questions, you must show work or give some explanation of your method. You must communicate to me **how** you reached your answer.

1. (**21 points**) Antiderivatives. Find each of the following.

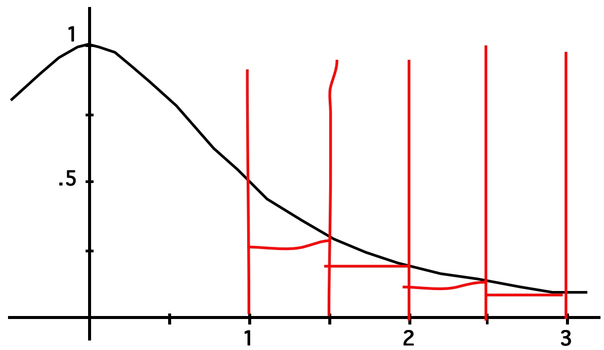
a. $\int 4x^3\, dx$ $= x^4 + C$

b. $\int e^{3x}\, dx$ $= \frac{1}{3}e^{3x} + C$

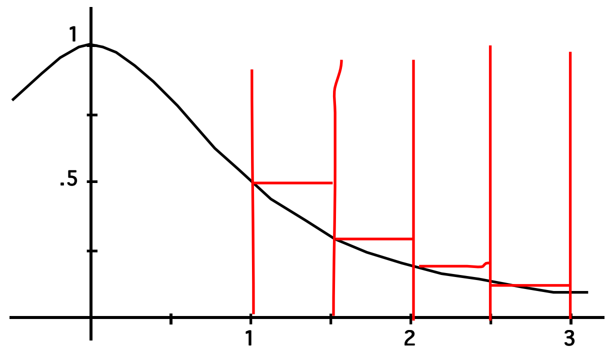
c. $\displaystyle\int \sqrt{x} - \frac{5}{x^2}\, dx$ $= \int x^{1/2} - 5x^{-2}\, dx$

$= \frac{2}{3}x^{3/2} + 5x^{-1} + C$

2. (**22 points**) Definite integral concepts. Both graphs below show the curve $y = \dfrac{1}{x^2+1}$ for $0 \le x \le 3$. Refer to these graphs as you answer the following questions about $\displaystyle\int_1^3 \frac{1}{x^2+1}\,dx$.



Under Estimate



Over Estimate

a. Compute a sum with four terms ($n = 4$) to approximate $\displaystyle\int_1^3 \frac{1}{x^2+1}\,dx$. Your sum should definitely be an **UNDER** estimate. Illustrate your sum by drawing rectangles on the left graph. Use the equation of the curve and show your work to compute a numerical value for this sum.

$\Delta x = \frac{3-1}{4} = \frac{1}{2}$ $\quad f(1.5)\frac{1}{2} + f(2)\frac{1}{2} + f(2.5)\frac{1}{2} + f(3)\frac{1}{2}$

$= \frac{1}{2}\left(\frac{1}{13/4} + \frac{1}{5} + \frac{1}{29/4} + \frac{1}{10}\right) = .373\ldots$

b. Compute another sum with four terms ($n = 4$) to approximate $\displaystyle\int_1^3 \frac{1}{x^2+1}\,dx$. This time your sum should definitely be an **OVER** estimate. Illustrate your sum by drawing rectangles on the right graph. Use the equation of the curve and show your work to compute a numerical value for this sum.

$f(1)\frac{1}{2} + f(1.5)\frac{1}{2} + f(2)\frac{1}{2} + f(2.5)\frac{1}{2} = .573\ldots$

Problem 2, continued

c. The average of your answers to a. and b. can used as an estimate for $\int_1^3 \frac{1}{x^2+1}\,dx$. This estimate is not exactly correct. At the worst, how far off might the estimate be from the exact answer? Explain how you reached your answer.

avg = .473 is .1 away from over & under estimates so error ≤ .1

d. Give a brief explanation of how the limit concept and sums like the ones above are used in the definition of $\int_1^3 \frac{1}{x^2+1}\,dx$.

Sums like the above give us over and under estimates for the integral. Taking more rectangles gives a better estimate, as we can see by noting the worst case error in each case. In the limit as the number of rectangles goes to infinity, the worst case error is 0. Therefore, the integral is equal to the (common) limit of the over and under estimates as the number of rectangles goes to infinity.

3. (**10 points**) Use the fundamental theorem of calculus (which links together derivatives and integrals) to find the exact value of the following integral: $\int_0^3 3x^2 - 2\,dx$ $= x^3 - 2x \Big|_0^3$

$= (27-6) - (0-0) = 21$

4. (**10 points**) Engineers have found the following equation for the rate at which a chemical dissolves in water:

$$R(t) = 8.32\left(\frac{1}{2}\right)^t \quad \text{grams per hour}$$

where t is in units of hours. This equation states, for example, that at time $t = 1$ hour, the chemical is dissolving at the rate of $8.32(1/2) = 4.16$ grams per hour, and similarly, at time $t = 2$ it is dissolving at the rate of $8.32(1/4) = 2.08$ grams per hour. Using the equation for $R(t)$, calculate the total amount of chemical that dissolved between time $t = 0$ and $t = 4$. Hint: You can find the answer using a definite integral. If you use the `fnint` operation on your calculator, be sure to write down on your paper what command you entered in the calculator.

total disolved $= \int_0^4 8.32\left(\frac{1}{2}\right)^t dt$

$= 8.32 \frac{1}{\ln 1/2}\left(\frac{1}{2}\right)^t \Big|_0^4 = \frac{-8.32}{\ln 2}\left(\left(\frac{1}{2}\right)^4 - \left(\frac{1}{2}\right)^0\right)$

$= \frac{8.32}{\ln 2}\left(1 - \frac{1}{16}\right) = \frac{8.32}{\ln 2} \cdot \frac{15}{16}$