

Applied Calculus 1
Spring 2000
Exam 3

Name _____

Instructions: This test has 7 problems, some with several parts, with a total of 100 points. For full credit on non-essay questions, you must show work or give some explanation of your method. You must communicate to me **how** you reached your answer.

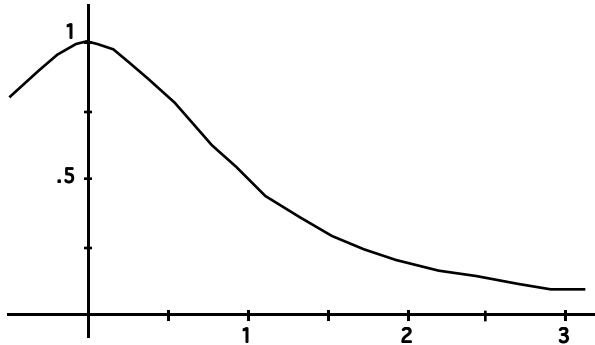
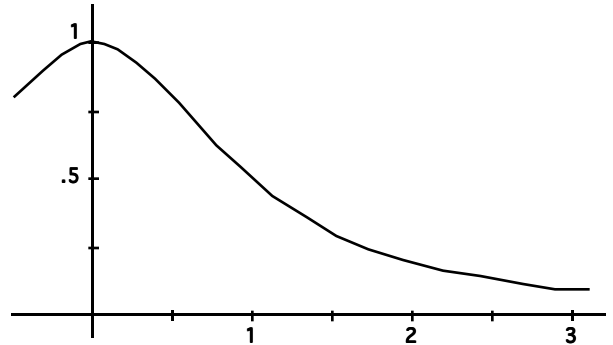
1. (**21 points**) Antiderivatives. Find each of the following.

a. $\int 4x^3 dx$

b. $\int e^{3x} dx$

c. $\int \sqrt{x} - \frac{5}{x^2} dx$

2. (22 points) Definite integral concepts. Both graphs below show the curve $y = \frac{1}{x^2 + 1}$ for $0 \leq x \leq 3$. Refer to these graphs as you answer the following questions about $\int_1^3 \frac{1}{x^2 + 1} dx$.

**Under Estimate****Over Estimate**

- a. Compute a sum with four terms ($n = 4$) to approximate $\int_1^3 \frac{1}{x^2 + 1} dx$. Your sum should definitely be an **UNDER** estimate. Illustrate your sum by drawing rectangles on the left graph. Use the equation of the curve and show your work to compute a numerical value for this sum.

- b. Compute another sum with four terms ($n = 4$) to approximate $\int_1^3 \frac{1}{x^2 + 1} dx$. This time your sum should definitely be an **OVER** estimate. Illustrate your sum by drawing rectangles on the right graph. Use the equation of the curve and show your work to compute a numerical value for this sum.

Problem 2, continued

- c. The average of your answers to $a.$ and $b.$ can be used as an estimate for $\int_1^3 \frac{1}{x^2+1} dx$. This estimate is not exactly correct. At the worst, how far off might the estimate be from the exact answer? Explain how you reached your answer.

- d. Give a brief explanation of how the limit concept and sums like the ones above are used in the definition of $\int_1^3 \frac{1}{x^2+1} dx$.

3. **(10 points)** Use the fundamental theorem of calculus (which links together derivatives and integrals) to find the exact value of the following integral: $\int_0^3 3x^2 - 2 \, dx$

4. **(10 points)** Engineers have found the following equation for the rate at which a chemical dissolves in water:

$$R(t) = 8.32 \left(\frac{1}{2}\right)^t \text{ grams per hour}$$

where t is in units of hours. This equation states, for example, that at time $t = 1$ hour, the chemical is dissolving at the rate of $8.32(1/2) = 4.16$ grams per hour, and similarly, at time $t = 2$ it is dissolving at the rate of $8.32(1/4) = 2.08$ grams per hour. Using the equation for $R(t)$, calculate the total amount of chemical that dissolved between time $t = 0$ and $t = 4$. Hint: You can find the answer using a definite integral. If you use the `fnint` operation on your calculator, be sure to write down on your paper what command you entered in the calculator.