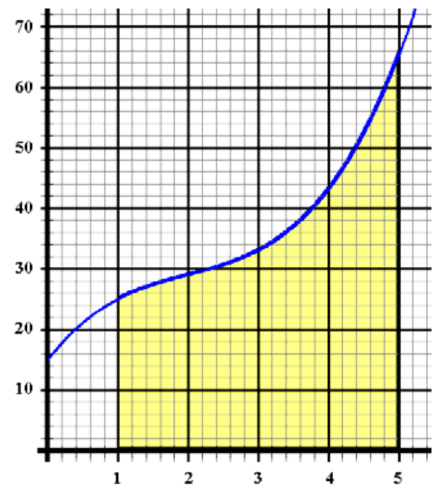
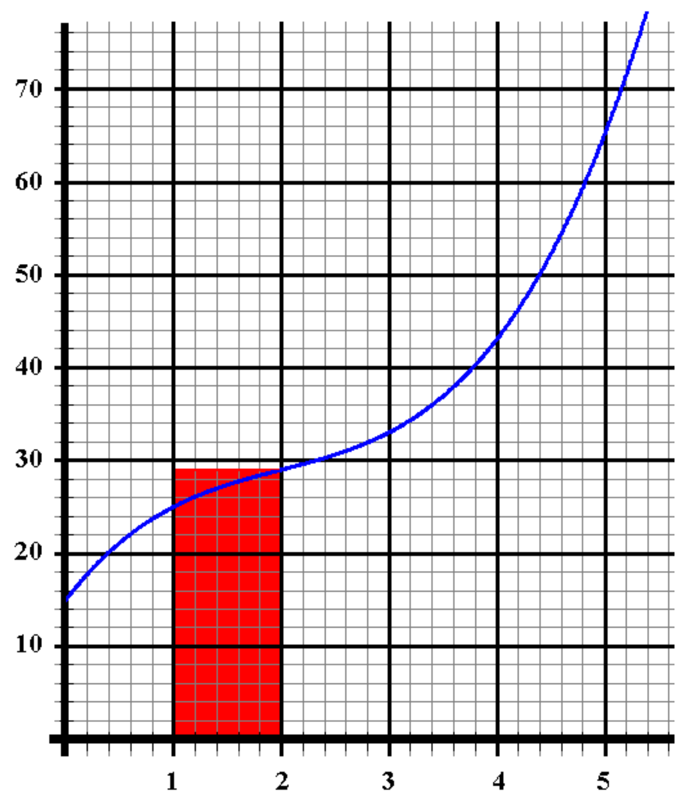
Calc 1 Worksheet: Area Under a Curve



The idea of accumulating change is closely related to the problem of finding areas under curves. To be more accurate, our goal is to find the area between the graph of a function $y = f(x)$ and the x axis between two specified values of x. The shaded region in the figure at right illustrates this idea, in this case showing the area between $x = 1$ and $x = 5$. ($f(x) = x^3 - 6x^2 + 15x + 15$.)

To begin your study of this topic, you will find estimates for the area using narrow rectangles. Using a collection of rectangles that completely covers up the shaded region gives an over estimate. Using rectangles that are completely contained within the shaded region gives an under estimate. Then we can refine these estimates using a process of successive approximation, similar to our approach for finding slopes of tangent lines.

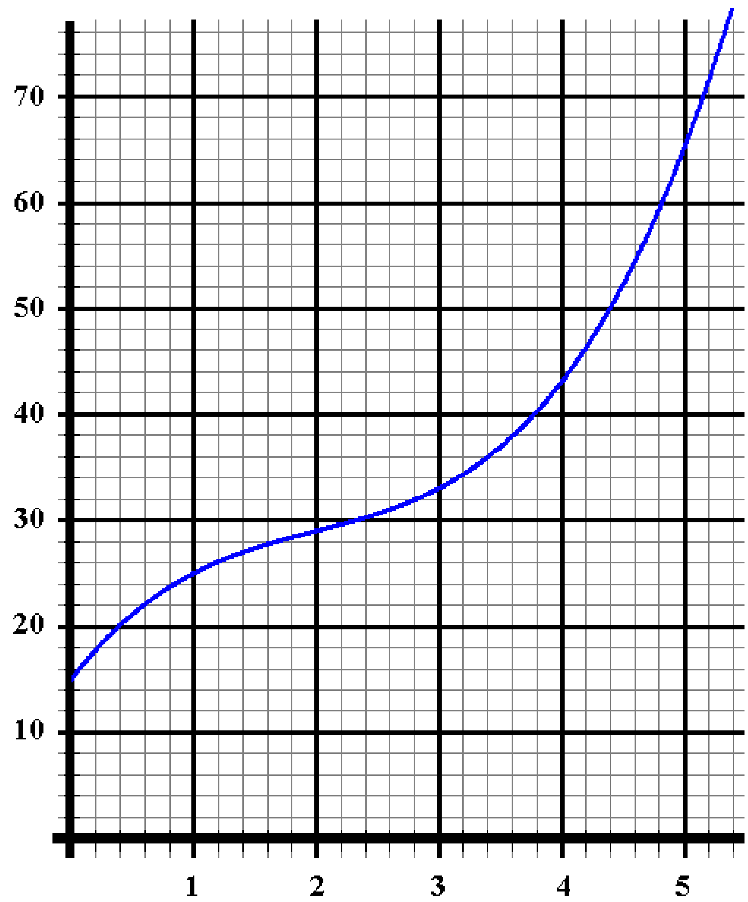


1. The figure at right shows a red rectangle. Notice that this rectangle completely covers up the part of the curve between $x = 1$ and $x = 2$. What is the area of this rectangle?

2. Draw three more rectangles on the figure, each one unit wide, and each standing on the x axis, so that together they cover up the curve between $x = 2$ and $x = 5$. For each one find the area.

3. Combining the areas of the four rectangles from parts 1 and 2 provides an estimate of the yellow region in the figure at the top of the page. Is it an over estimate or an under estimate for the yellow area? How can you tell?

4. Use the graph at right and a similar method as before to find another approximation with four rectangles. This time, make sure all of the rectangles are completely contained within the area we are estimating. Draw your rectangles on the graph and find their areas below.

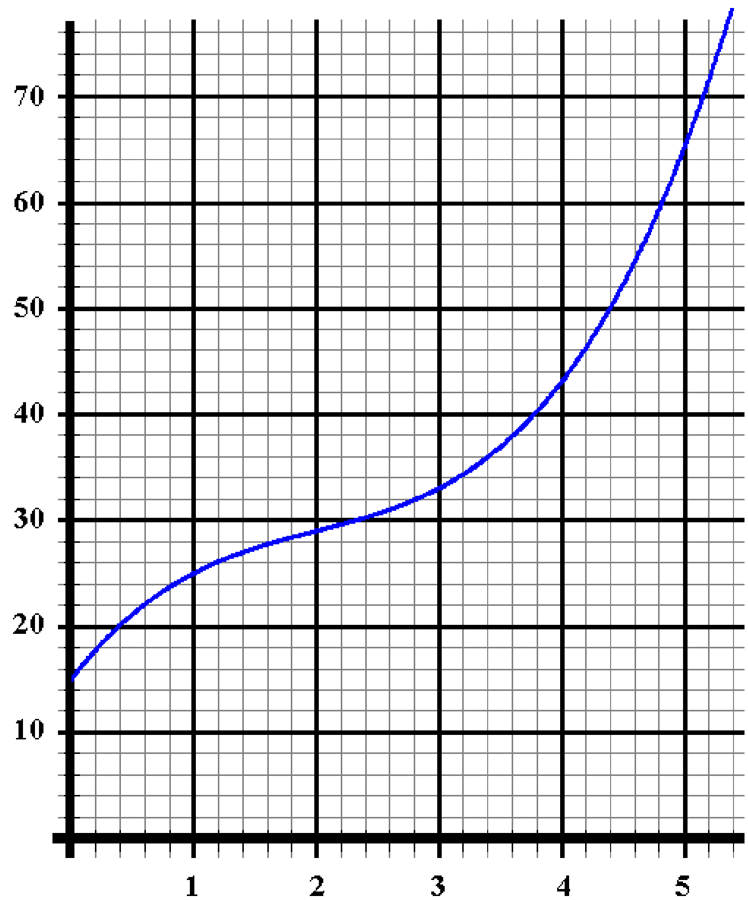


5. Total the areas of your four rectangles to obtain an estimate for the yellow area in the first figure on page 1.

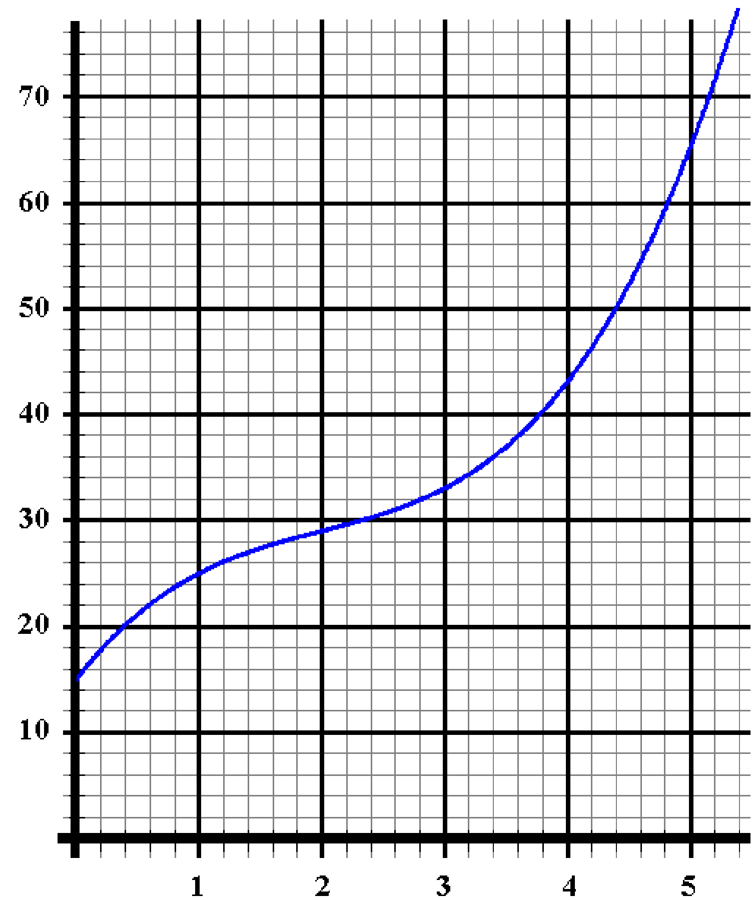
6. Is your answer from part 5 an over or under estimate? How can you tell?

7. At this point you should have two estimates of the yellow area. One is too low and the other is too high. We know the true area is in between the two estimates. This suggests using the average of the two estimates as an approximation to the true area. At most, how large an error would be possible for this approximation? Why?

8. On this page and the next, repeat the methods from the first two pages, but this time use rectangles that are 1/2 unit wide rather than 1 unit wide. On this page use rectangles that completely cover up the yellow region from page 1. Draw the rectangles on the graph, and compute the areas below. Then add all the areas together to find an estimate for the area of the yellow region. Is it an over or under estimate of the true area? Why?



9. Next use rectangles that are completely contained within the yellow region from page 1. Draw the rectangles on the graph, and compute the areas below. Then add all the areas together to find an estimate for the area of the yellow region. Is it an over or under estimate of the true area? Why?



10. As before, use the average of your over- and under-estimate to approximate the true area of the yellow region. Arguing as before, what is the largest possible error from this new approximation? Comparing the largest error here with the one found earlier, which approximation is the more accurate?