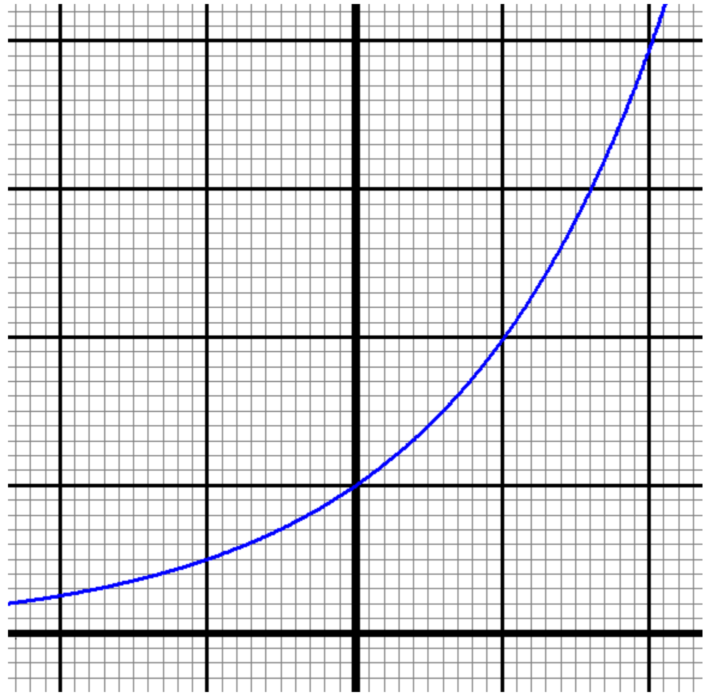
Calc 1 Worksheet 1: Finding a Tangent Line



Part 1: Using Successive Approximation

The figure at right shows the graph of the function $f(x) = 2^x$. Using a straightedge, draw a tangent line as accurately as possible at the point (1,2). Then draw a secant line through the points (1,2) and (2,4). We will refer to that as the *right* secant line. Draw another secant line, this time through the points (-1, .5) and (1,2). This is the *left* secant line. Answer the following questions.

1. What is the slope of the left secant line?

2. Based on the graph, is the slope of the left secant line greater than, equal to, or less than the slope of the tangent line?

3. What is the slope of the right secant line?

4. Based on the graph, is the slope of the right secant line greater than, equal to, or less than the slope of the tangent line?

5. Using the results above, fill in the blanks with numerical values:

   _________ < slope of tangent line < ___________

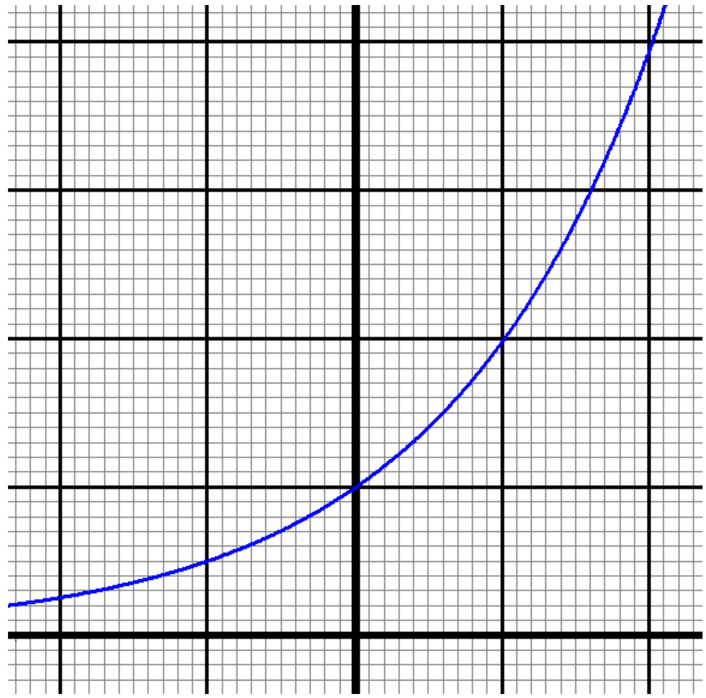
6. Based on the foregoing, what is your best numerical estimate of the tangent line, and how far off might this estimate be, at worst?

Improving the estimate.

Repeat the process you have just gone through, but this time use points that are closer to (1,2) to define your secant lines. Find the coordinates of your points accurately using a calculator and the equation $f(x) = 2^x$. As before, your goal is to fill in the blanks in the statement

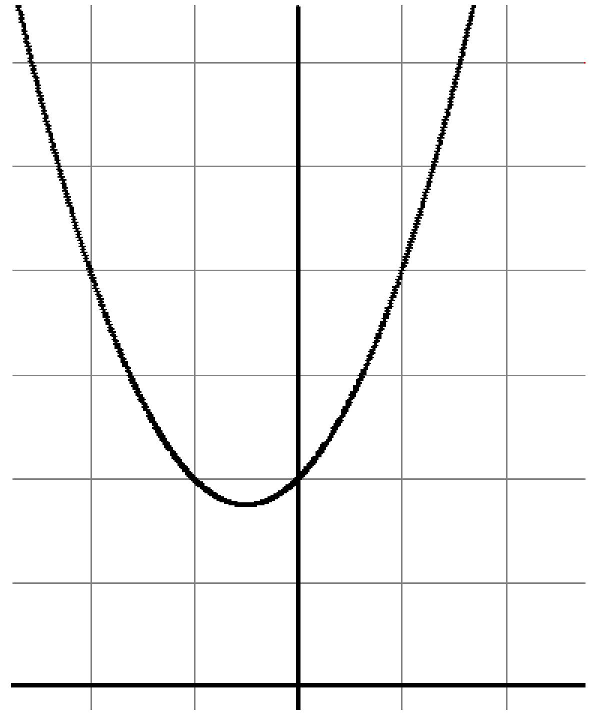
________ < slope of tangent line < ________

but this time, with estimates that are closer to the true value than you found before. Show your work and at the end, give an improved estimate for the slope of the tangent line, again indicating how far off your estimate might be at worst.



Part 2: Verifying an Exact Tangency

Algebraic methods can be used to verify exact tangency for simple curves. To illustrate, follow the steps below to find the tangent line for the curve $y = x^2 + x + 2$ at the point (1,4).



1. On the graph at right, it appears that the point (1,4) is on the graph. Verify that this is correct using the equation of the curve.

2. Using a straightedge, carefully draw a line that appears to be tangent to the curve at (1,4). Estimate the slope by finding the coordinates of two points on your line.

   Points ______________________________ Slope __________________________________

3. Using the slope you found, write the equation of your line. Hint: use the point-slope form of the equation of a line, (y - ♥) = m (x - ♣), where ♣ and ♥ are the coordinates of one point of the line and m is the slope.

4. Solve for y in your equation, and graph it with a graphing calculator or computer (*eg* with www.desmos.com/calculator). Also enter $y = x^2 + x + 2$ and graph that in the same window. Zoom in to see if your line seems to be a tangent line. If it does not, try changing the slope to find what does seem to be a tangent line. When you are done, record below the equation of a line that is your best result for a tangent line. Then continue to the other side of this sheet.

5. To verify that your line is truly tangent to the curve, you can use algebra to see whether your line and the curve intersect in more than one point. From the way you defined the line, you know it goes through (1,4) and that is also on the curve. To see whether there are any other points of intersection, combine the equations of your line and the given curve and solve for *x*. Show all of your work below.

6. Is your line truly a tangent line? If not, try to modify the equation of the line so that it WILL be exactly tangent. When you do have the exact tangent line, something interesting happens when you use algebra to find the points of intersection of the line and the original curve. What is it?