Elementary Math Models
Spring 2015
Exam 1 Name _____

The following formulas might be useful on this exam.

$$a_{n+1} = a_n + d \qquad y = mx + b \qquad (y - y_0) = m(x - x_0)$$

$$a_n = a_0 + dn \qquad \frac{x}{a} + \frac{y}{b} = 1$$

Instructions: Read the questions carefully, and be sure to answer all parts of each question. For full credit on non-essay questions, you must show work or give some explanation of your method. You must communicate to me **how** you reached your answer.

1. **[25 points]** Give brief definitions for each of the following terms (this page and the next). Each answer should explain to someone who has never studied the material in this course what each term **is**. Also, each answer can *include* an example, but should be more than just an example.

a. recursion or recursive method

b. position number

c. functional equation for a sequence

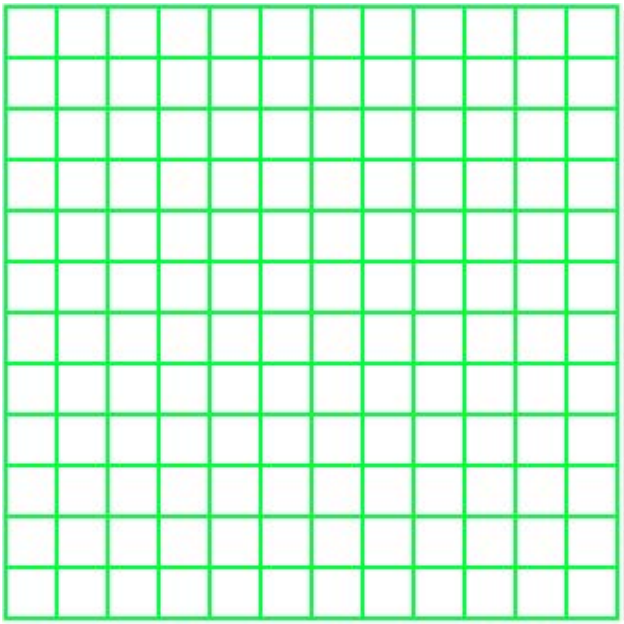
d. arithmetic growth

e. slope of a line

This is from section 2.3 and is not covered on our exam 1.

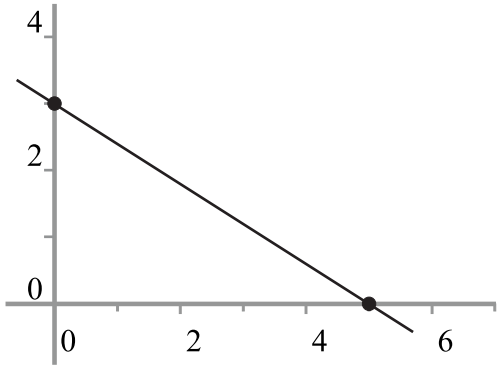
2. Math Skills [**30 points**] (This page and the next two)

   a. On the graph below draw a straight line that goes through the point $(-4, 1)$ and has a slope of $5/4$. Draw in and label your axes. To the right of the graph, give the equation of the line.



The items on this page are from section 2.3 and are not covered on our exam 1.

   b. Find an equation for the line shown in the graph below. Show your work or explain your method.



c. A number pattern is given by: $3, 10, 17, 24, 31, \cdots$. Denote the terms of this pattern using subscript notation, as follows: $a_0 = 3$, $a_1 = 10$, and so on. Find a_5 and a_{1000}. Show work or explain how you found your answers.

d. A number pattern starts with $a_0 = 10$ and obeys the difference equation $a_{n+1} = a_n + 5 - n$. Find a_1 and a_2. Show work or explain how you found your answers.

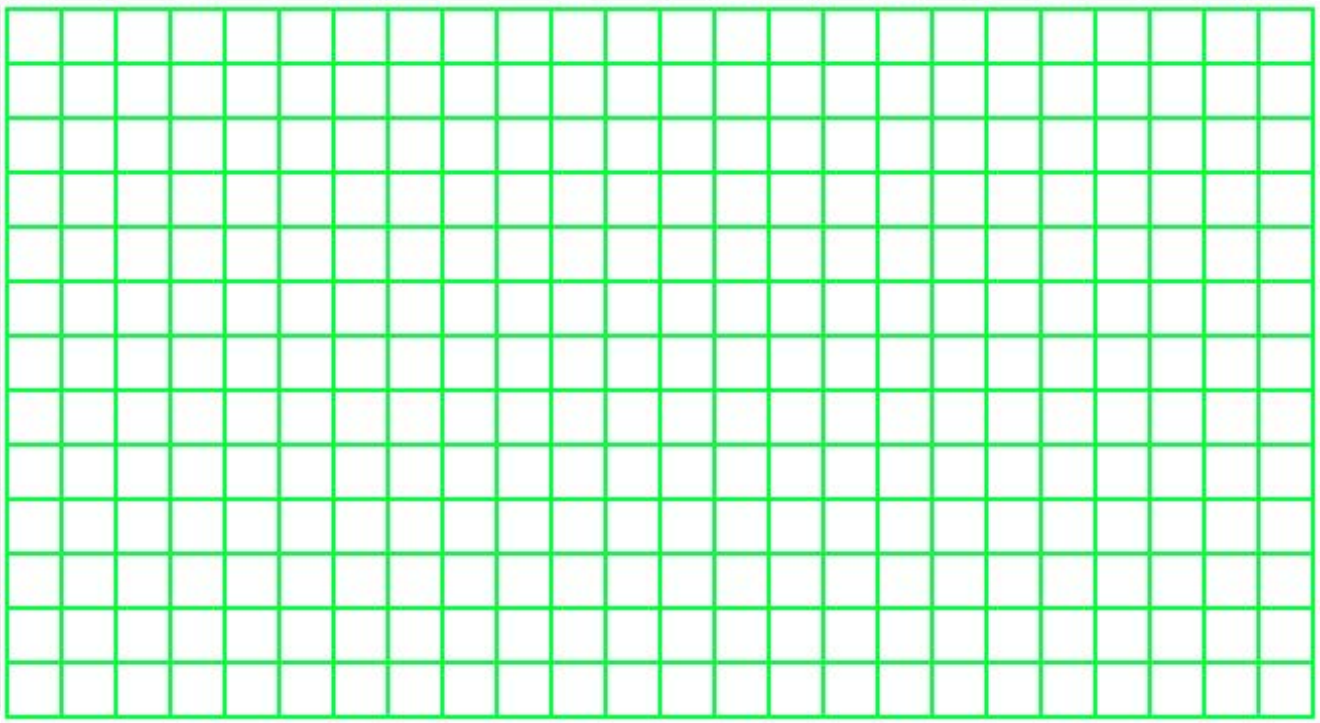
e. Solve this equation for x: $500 - 28x = 150$. Show all your steps, and verify that your answer is correct.

3. **[30 points]** A county board of education has been gathering data on home schooling for several years. The data show that in 2007 there were about 3200 home schooling families, in 2008 there were 4500, and in 2009 there were 5800. Make up an arithmetic growth model for this situation. Include the following in your answer:

- A data table
- Definitions for your variables
- A difference equation for your model
- A functional equation for your model
- A graph for your model (use the grid provided on page 7)
- A prediction, based on the model, of the number of home schooling families in 2018 (plus the work that justifies this prediction)
- A prediction, based on the model, of the year in which the number of home schooling families will reach or exceed 50,000 (plus the work that justifies this prediction)

Additional work space is provided on the next page.

Additional workspace for problem 3.



Grid for your graph. Be sure to label your axes.

Part II. Multiple Choice. [15 points] Circle the correct response for each item. You may write a brief explanation if you wish, but are not required to do so.

1. Which verbal description fits the following sequence? $0, 1, 3, 6, 10, 15, 21, \cdots$.
   (Assume 1 is the position number for the first term.)

   a. These terms are multiples of three.

   b. If you double any term and then add one, you'll get the next term.

   c. If you add the position number to any term, you'll get the next term.

   d. This is arithmetic growth with a starting term 0 and an added amount of 3 at each step.

2. Which equation fits the sequence described below?

   Three times the previous term plus 2 produces the next term.

   a. $a_n = 3n + 2$ b. $a_n = 3a_{n-1} + 2$ c. $a_{n+1} = 3 + 2a_n$ d. $a_{n+1} = 3a_{n+2}$

3. For a sequence $9, 3, 15, 23, 19, 17, \cdots$, if $a_n = 3$ then

   a. $a_{n+1} = 15$ b. $a_{n+1} = 23$ c. $a_{n+1} = 4$ d. $a_{n+1} = 16$

4. Which of the following would **not** indicate arithmetic growth of a number sequence?

   a. When plotted on a graph, the data appear to lie on a straight line.

   b. The terms are given by a functional equation of the form $a_n = a_0 + dn$.

   c. Increases between successive terms follow the pattern of even numbers $2, 4, 6, 8, \cdots$.

   d. The terms can be generated recursively by repeatedly adding a specific constant.

5. Which of the following is most correct?

   a. An arithmetic growth sequence should be used in a model only if a graph of the data shows points that fall exactly on a straight line.

   b. An arithmetic growth sequence may be used if the differences between data points are approximately constant.

   c. Arithmetic growth sequences are rarely used in real applications.

   d. In arithmetic growth models, proportional reasoning should not be assumed.