

Part I. Multiple Choice. [24 points] Circle the correct response for each item. You may write a brief explanation if you wish, but are not required to do so.

1. An epidemiologist is developing a model for the spread of measles during a recent outbreak in the US. In the model M_n is the number of people who have been infected with measles by the end of week n , and the following functional equation is found:

$$M_n = 10,000 + 846n.$$

Which statement most correctly describes this equation?

- It expresses M_n as a function of n .
 - It expresses n as a function of M_n .
 - It expresses M_n as a quadratic function of n .
 - It is an example of the quadratic formula.
2. A quadratic growth sequence has constant second differences all equal to 25. If $a_1 = 1000$ and $a_5 = 5000$, which of the following must be true?

- a. $a_9 = 9,000$ b. $a_9 > 9,000$ c. $a_9 < 9,000$ d. Can't tell from the given information.

3. Which of the following is NOT a valid reason for adopting a quadratic growth model?
- The differences of the differences form a simple pattern such as 3, 6, 9, 12, 15, 18, \dots .
 - The second differences are nearly, but not exactly, equal.
 - A recursive analysis shows that the difference between a_n and a_{n+1} is a linear function of n .
 - We are interested in the running totals of an arithmetic growth sequence.
4. In a model of social media networks, two participants who are “friends” are said to share a friendship link. In the model, the number of friendship links is given as a quadratic function of the number of participants. If the number of participants is doubled in the model, which the of following is most likely to occur?
- The number of friendship links will approximately double.
 - The number of friendship links will exactly double.
 - The number of friendship links will not change.
 - The number of friendship links will approximately quadruple (that is, increase by a factor of four).

The following equations might be useful on this exam.

$$1 + 2 + 3 + \cdots + n = n(n + 1)/2$$

$$a_{n+1} = a_n + d$$

$$a_{n+1} = a_n + d + en$$

$$y = mx + b$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$a_n = a_0 + dn$$

$$a_n = a_0 + dn + e \left(\frac{(n-1)n}{2} \right)$$

$$y - y_0 = m(x - x_0)$$

Part II: Free Answer. Read the questions carefully, and be sure to answer all parts of each question. For full credit on non-essay questions, you must show work or give some explanation of your method. You must communicate to me **how** you reached your answer.

1. Essay Questions on Quadratic Growth [**20 points**]. Write short essay responses to the items on this page and the next. Your answers should be written in one or more complete English sentences. Each answer can *include* an example, but there should also be an explanation of the meaning and significance of any examples you use.

a. Explain how to recognize quadratic growth in a number sequence (or number pattern).

b. Is proportional reasoning valid for quadratic growth models? Explain why or why not.

- c. What is meant by running totals for a number sequence, and what does this topic have to do with quadratic growth?
- d. In class we discussed the handshake problem, where the king must shake the hand of each guest three times, and each guest has to shake the hand of every other guest once. Let h_n be the total number of handshakes that must occur at an event with the king and n guests. Use a recursive analysis to find a difference equation for h_n , explaining your reasoning.

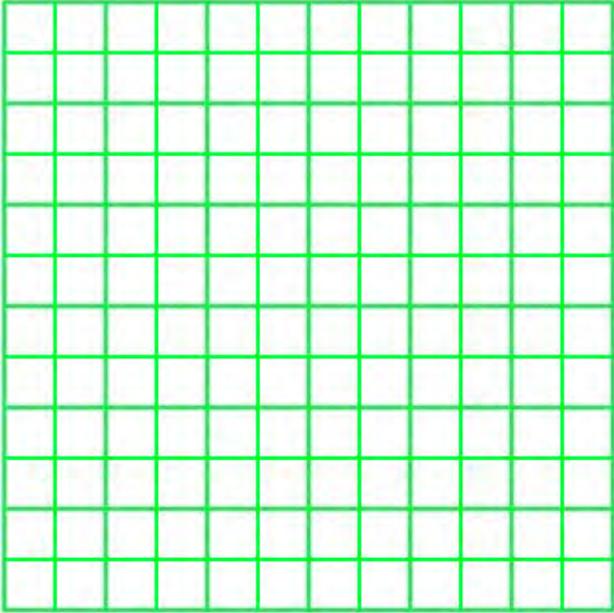
NOTE: These questions only concern quadratic growth. Also be prepared to show that you understand terms from sections 2.3 and 2.4, such as slope, intercept, and proportional reasoning.

2. Math Skills [28 points]. Solve the problems on the this page and the next. You must **show work** for full credit.

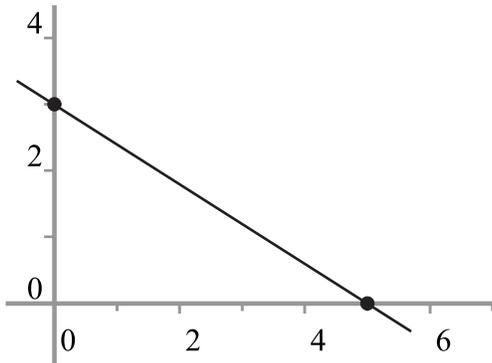
- a. A number sequence has difference equation $a_{n+1} = a_n + 3 - 2n$ and the starting value is $a_0 = 12$. Use the difference equation to find a_1 and a_2 . Show your work.

- b. A number sequence has difference equation $b_{n+1} = b_n - 3 + 4n$ and a starting value of $b_0 = 24$. Find the functional equation and use it to find b_{20} .

- c. On the graph below draw a straight line that goes through the point $(-4, 1)$ and has a slope of $5/4$. Draw in and label your axes. To the right of the graph, give the equation of the line.



- d. Find an equation for the line shown in the graph below. Show your work or explain your method.



4. Modeling Problem [**20 points**] (this page and the next). A research team is studying the growth of traffic in a suburban area. Based on their data and some analysis, they have estimated the average daily number of miles driven by each car in the area each year for the past several years. Here are their data:

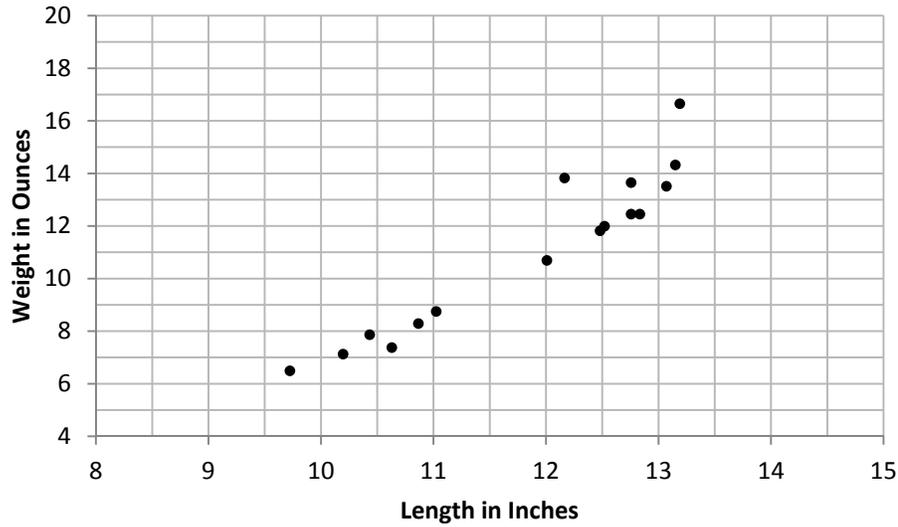
Year Number	0	1	2	3	4
Average Miles Driven per day	12.5	15.0	18.0	21.5	25.5

- a. What kind of growth model would be appropriate for this problem? Why?
- b. Develop an appropriate model for the data. Define the variables that you use, and give both a difference equation and a functional equation, clearly labeled.

c. According to your model, how many miles will be driven per day, on the average, in year 5?

d. According to your model, how many miles will be driven per day, on the average, in year 20?

5. [8 points] In the graph below each point represents the weight and length of one fish taken from a population in Washington state.



- Using a straight edge, draw a straight line that fits the data as closely as possible.
- Find an equation of the line you drew in part a, expressing weight W as a function of length L . Show work and/or explain your method.
- Using your equation as a linear model for the data, about what length would you predict for a fish that weighs 12 ounces?