Elementary Math Models

Repeated Drug Dose Mixed Model

This worksheet explores a mixed model for the accumulation of a drug in the body as a result of a regular schedule of repeated doses. All of the examples we work with will involve variations of this difference equation:

$$a_{n+1} = \frac{3}{4}a_n + 100$$

The instructions refer to an excel spreadsheet for easily generating numerical data and graphs for models. If you use this spreadsheet you can easily enter the difference and functional equations for the models.

1. The medicine dosage model. A patient is given a dose of medication every four hours. It is known that in four hours, the body will remove approximately 1/4 of this drug from the blood stream. Suppose that the patient is initially given 500 units of the medicine, and then an additional 100 units every 4 hours. After the first dose is taken, there are 500 units of the drug in the blood stream. At the end of four hours, 1/4 of that has been removed, so that only 375 units remain. Then another 100 units are taken, boosting the amount of drug in the blood to 475. After another 4 hours, 1/4 of the 475 is removed, leaving 3/4 of 475, or 356.25, then 100 units are added, for a total of 456.25 units of the drug in the blood stream.

We can develop a difference equation model for this situation. We will define the variable n to be the number of doses of medication taken, and a_n to be the amount of drug in the blood immediately after the nth dose is taken. For example a_0 is the amount of drug immediately after the starting dose, a_1 the amount immediately after the first repeated dose, and so on. Then the situation described above is represented by the following difference equation and initial value:

$$a_{n+1} = (3/4)a_n + 100; \quad a_0 = 500.$$

Use the excel spreadsheet to examine the pattern of numerical values and the graph for this model. Does the amount of drug keep building up, or does it eventually level off? Write a brief answer and an explanation here.

2. You should have found in part 1 that the amount of drug in the blood levels off to about 400 units. In this section you will investigate variations of the model. First, keeping everything else about the problem the same, you will try some different values for the initial dosage. Remember that the repeated dosage is supposed to stay the same, 100 units. So the difference equation will stay the same. Just the starting point, a_0 is supposed to be changed. Observe using the spreadsheet what happens when a_0 is each of the following values: 100, 200, 300, 400, 600, 700, and 800. For each of these values, the spreadsheet will show the first 25 data points and a graph. From what you see, answer these questions: Does the amount of drug in the blood still level off? If so, to what? About how long does it take to level off? What general conclusion can you reach about how the initial dose affects the level of drug in the blood over the long term?

Now suppose that the initial dosage is kept fixed at 500 units, but the repeated dosage is changed. Instead of giving the patient 100 units every four hours, what if 50 units are given? 20 units? 200 units? 500 units? How does that change the difference equation? If the repeated dosage is 50 units, for example, the difference equation will be

$$a_{n+1} = (3/4)a_n + 50$$

Use the spreadsheet to see what happens for this difference equation (using $a_0 = 500$). Then repeat the problem using each of the following values for the repeated dosage: 20, 200, 500. Enter the results in the following table:

Repeated Dosage Amount	Level Off Amount
20	
50	
100	400
200	
500	

Can you see a pattern that relates the size of the repeated dosage with the amount at which the drug eventually levels off? Use this pattern to predict where the model will level off if the repeated dosage is

1000 units. Check your prediction with the spreadsheet. Write a paragraph about your findings below.

Based on your findings, what should the patient be given as a repeated dosage if you want the drug to level off to about 760 units?

3. In the preceding variations, the amount of drug removed from the body between doses was always kept at 1/4. For different drugs, or for different lengths of time between doses, this fraction can change. Now you will investigate the effects of changing this parameter.

Observe that if 1/4 of the drug is removed, 3/4 remain, and that the 3/4 shows up in the difference equation

$$a_{n+1} = (3/4)a_n + \text{repeated dose}$$

For convenience we will express 3/4 in decimal form as .75

$$a_{n+1} = .75a_n + \text{repeated dose}$$

If the body removes 20% of the drug between doses, that leaves 80% in the body, so the difference equation becomes

$$a_{n+1} = .80a_n + \text{repeated dose}$$

You will investigate how this change affects the model, by repeating exercises from before using .80 in place of (3/4). Use the results to complete this table:

Repeated Dosage Amount	Level Off Amount
20	
50	
100	
200	
500	

4. You will now repeat the previous page but this time suppose that the body removes 40% of the drug between doses. After you complete that investigation, you'll look for a pattern connecting the percentage of drug removed from the body between doses (that is, the 25%, 20%, and 40% of the examples), the repeated dose amount, and the level amount. It may help to complete the table below. For the last line of the table, fill it in based on a pattern in the table.

Percent of Drug Removed Between Doses	Rule for Finding the Level Amount
25	Multiply Repeated Dose × 4
20	
40	
10	

Use your entry from the last line of the table to predict what the level amount will be with repeated doses of 50 units assuming that the body removes only 10% of the drug between doses. Use the spreadsheet to see if the prediction is correct. Remember that the difference equation will be

$$a_{n+1} = (.90)a_n + 50$$

5. Using the Equilibrium Equation. In the reading, we saw that in any mixed growth sequence, we can compute the equilibrium amount as $E = \dfrac{d}{1-r}$. Verify that this equation gives the same results as you found in the examples above.

Earlier, assuming $r = 0.75$, you were asked to specify a repeated dosage for which the drug woult level off at 760 units. That is the same as saying we want E to be 760. Thus, if we specify E and r, the point is to find d. To do so algebraically, use the equation $E = \dfrac{d}{1-r}$, substituting 760 for E and 0.75 for r. Solve for d. Do you get the same result you found earlier?