EMM **Worksheet: Logistic Growth**

This worksheet discusses 5 sample models that will give you some practice working with the ideas of logistic growth.

The basic framework for each model is the same. A laboratory is developing a procedure to grow a certain kind of mold that will be used to make a new antibiotic. The mold is grown in a vat with a nutrient solution made up of sugars, water, and other ingredients. For each different nutrient solution, the laboratory makes two tests by introducing a known amount of the mold and observing how the mold grows over a 24 hour period. For each of the two test amounts, the growth factor for the 24 hour period is recorded. The results of these tests are shown in the table below.

Test	First Test		Second Test	
Solution	Population	Growth Factor	Population	Growth Factor
A	100	0.8	500	0.6
B	100	0.9	400	0.3
C	100	2.0	500	1.2
D	100	3.0	600	1.5
E	100	4.0	600	1.5

For each Test Solution (A - E), your job is to develop and analyze a logistic growth model. A special worksheet has been provided for this purpose. Use a separate worksheet for each model, and follow the outline below.

Three copies of the worksheet are attached. You can get additional copies at the webpage for the course.

1. Develop a linear equation that relates the growth factor to the population size. There is room on the work sheet to graph the line and work out the equation.
2. Formulate a difference equation in the form $p_{n+1} = m(L - p_n)p_n$. What are the constants m and L for this model?
3. Make a graph using p_n as the x value and p_{n+1} as the y value. Find the highest point on the graph. Use this to decide whether the model will always produce values of p_{n+1} that are between 0 and L for your L. Relate your conclusion to the condition $mL < 4$.
4. Is there a value of the population that results in a growth factor equal to 1? What does that tell you about the future population growth for this model? Relate your conclusion to the value of $L - 1/m$.
5. Verify your conclusions by computing and graphing the first several values of population model for some starting population.

Complete an attached worksheet (both sides) for at least 3 of the 5 test solutions. We will complete one as a class, and you may have time for a second during class. Others can be completed for homework.

After you have finished completing the worksheets, go on to the instructions on the reverse of this page.

Elementary Math Models

Another Worksheet for Logistic Growth

Do the problems for this worksheet on a separate sheet of paper.

Summary and expansion of discussion in the book: In a logistic growth model, calculate mL. Then one of the following cases must apply:

- If $mL \le 1$ the population will die off for any initial population size between 0 and L.
- If $1 < mL \le 3$ the population will level off eventually, for any initial population size between 0 and L. The steady population size will be $L - 1/m$.
- If $3 < mL \le 4$, the population may not level off. However, you can be sure that the model will always produce population values that stay between 0 and L, provided the initial population size is between 0 and L.
- If $mL > 4$, the model may eventually produce negative values for population size. This is invalid and shows that the model is unreliable for future predictions.

In the first logistic growth worksheet, you were supposed to develop logistic growth models for growing mold in a laboratory under a variety of conditions. The difference equations that you should have found are shown below. For each one, use the results of the logistic growth chapter to decide if the population eventually dies out, levels off, remains valid without leveling off, or eventually leads to negative (and so invalid) predicted population sizes.

a. $p_{n+1} = 0.0005(1700 - p_n)p_n$

b. $p_{n+1} = 0.002(550 - p_n)p_n$

c. $p_{n+1} = 0.002(1100 - p_n)p_n$

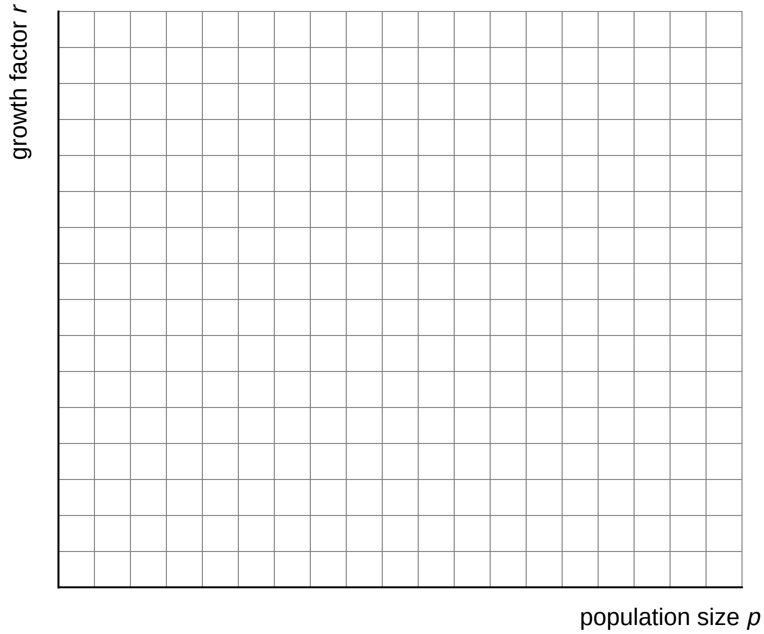
d. $p_{n+1} = 0.003(1100 - p_n)p_n$

e. $p_{n+1} = 0.005(900 - p_n)p_n$

Logistic Model for Mold Growth Experiment

Nutrient Test Solution ____________

1. Find the equation for r as a function of p.



2. Find thedifference equation for this population model in the form $p_{n+1} = m(L-p_n)p_n$.

m = _______

L = _______

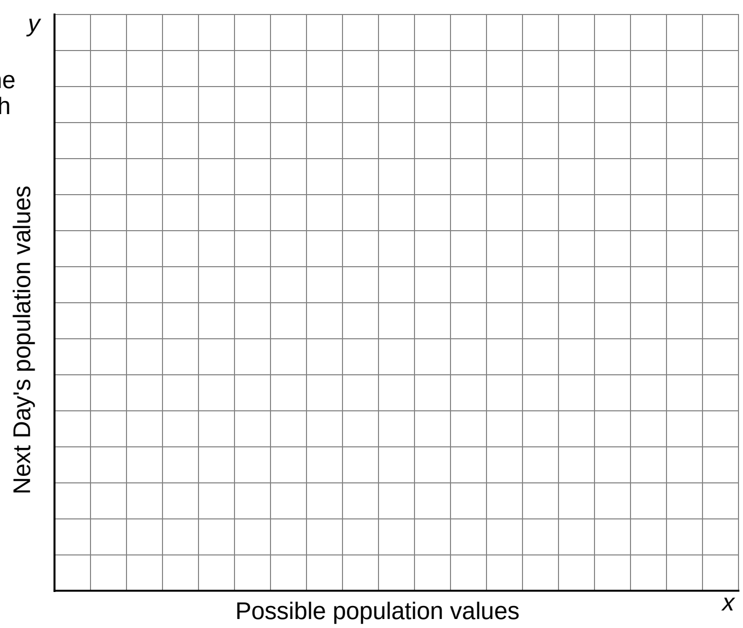
3. Graph using x for any possible population size, and y as the population size one day later. Write the equation for the graph here:

y = ______________________

x intercepts ____________________

axis of symmetry __________________

High point ________________________

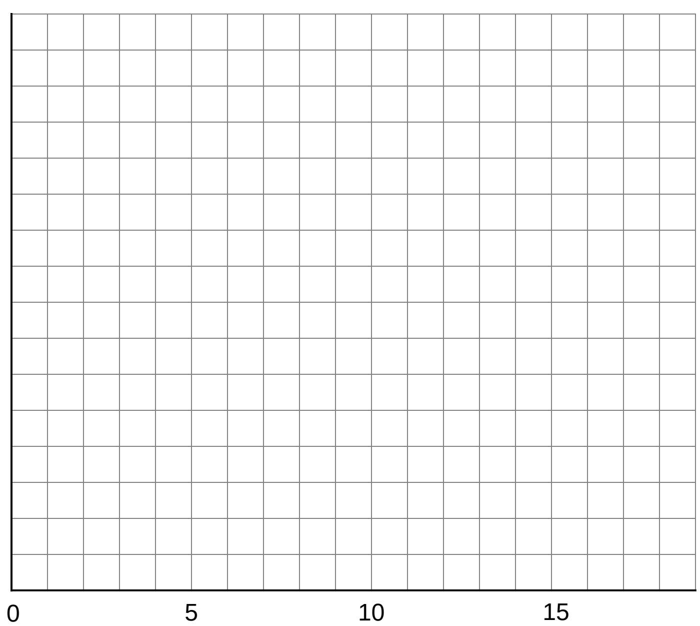


4. Fixed Point Calculation. Use your equation for *r* and *p* to find a value of *p* that makes *r* equal to 1 and write the answer below

$p =$ ___________ $L-(1/m) =$ __________________

Could p_n ever reach the value you found above?___________   If so, what will p_{n+1} be?

5. Testing the model. Based on your work above, what do you think will happen with this model? Will the model ever lead to negative values for the population? Will it eventually level off? Will the population just go down to 0, indicating that all the mold will simply die off? Write a short explanation below.



Day (*n*)	Population size (p_n)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Logistic Model for Mold Growth Experiment

Nutrient Test Solution ____________

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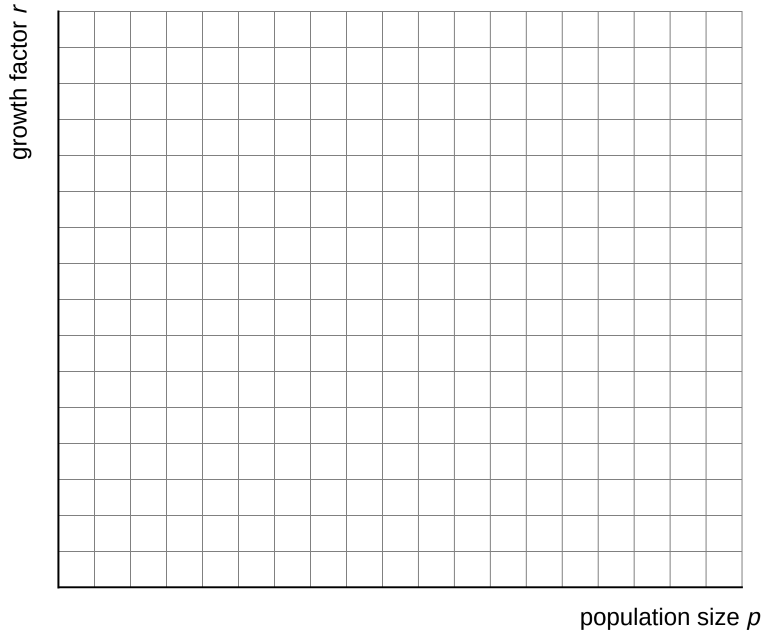

2. Find thedifference equation for this population model in the form $p_{n+1} = m(L-p_n)p_n$.

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3. Graph using x for any possible population size, and y as the population size one day later. Write the equation for the graph here:

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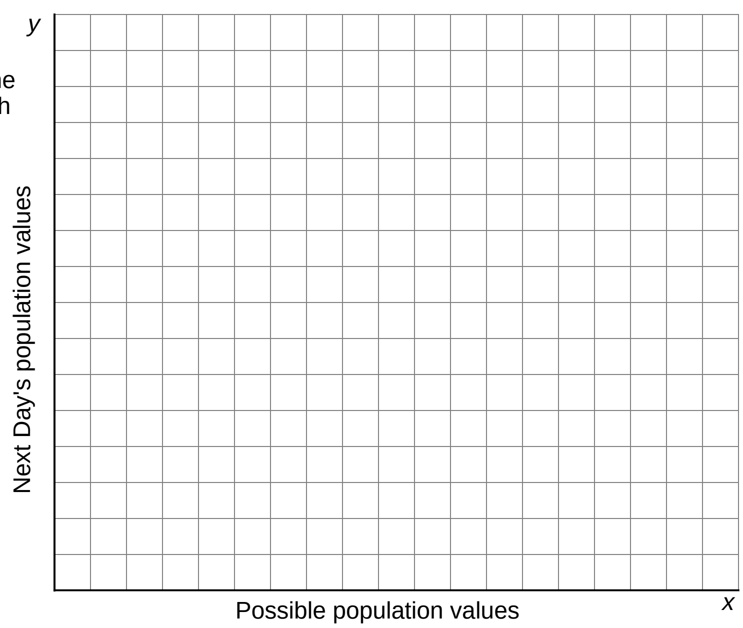
axis of symmetry __________________

High point _________________________

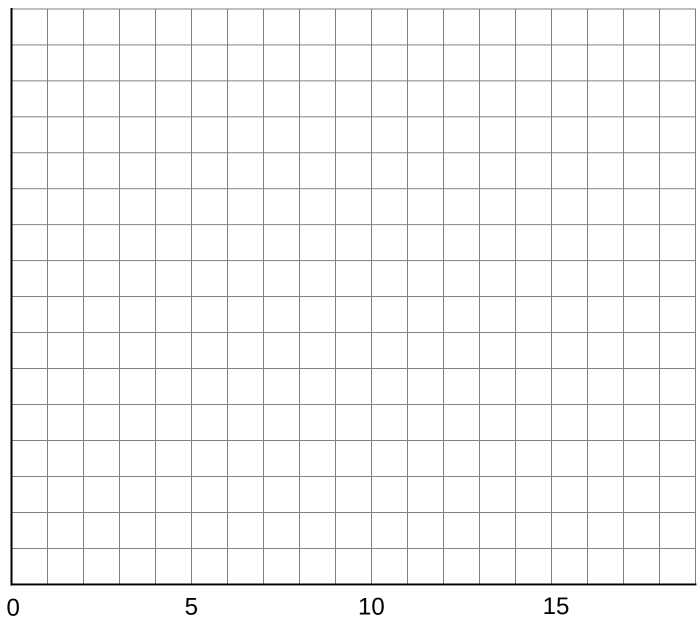


4. Fixed Point Calculation. Use your equation for r and p to find a value of p that makes r equal to 1 and write the answer below

p =__________ $L-(1/m)$ = ________________

Could p_n ever reach the value you found above?__________ If so, what will p_{n+1} be?

5. Testing the model. Based on your work above, what do you think will happen with this model? Will the model ever lead to negative values for the population? Will it eventually level off? Will the population just go down to 0, indicating that all the mold will simply die off? Write a short explanation below.



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Logistic Model for Mold Growth Experiment

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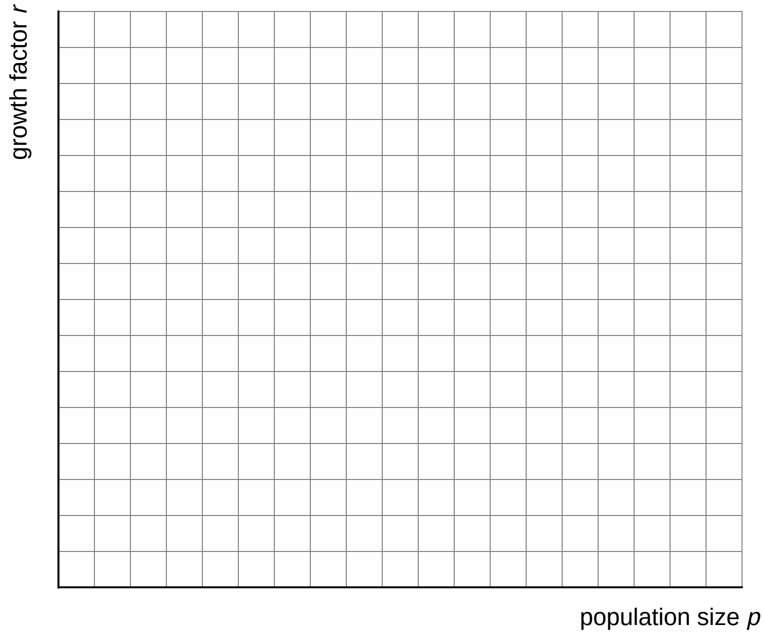

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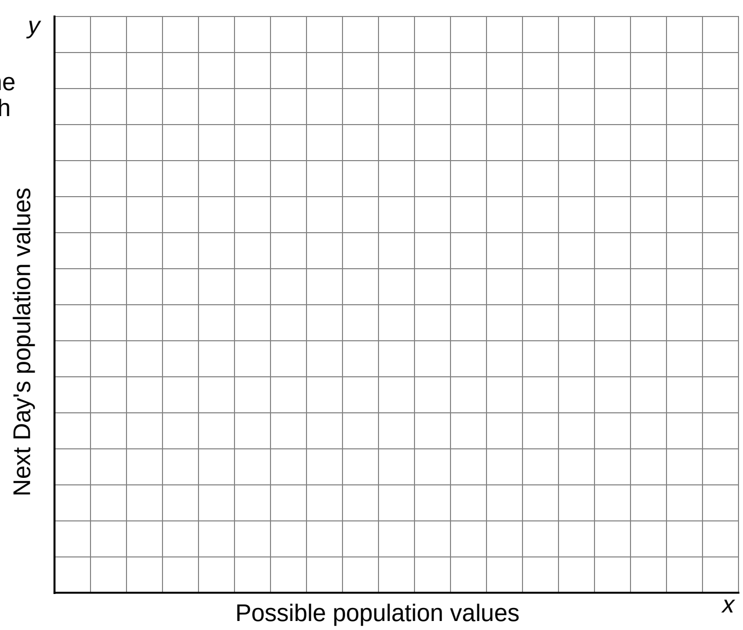

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p =___________ $L-(1/m)$ = _________________

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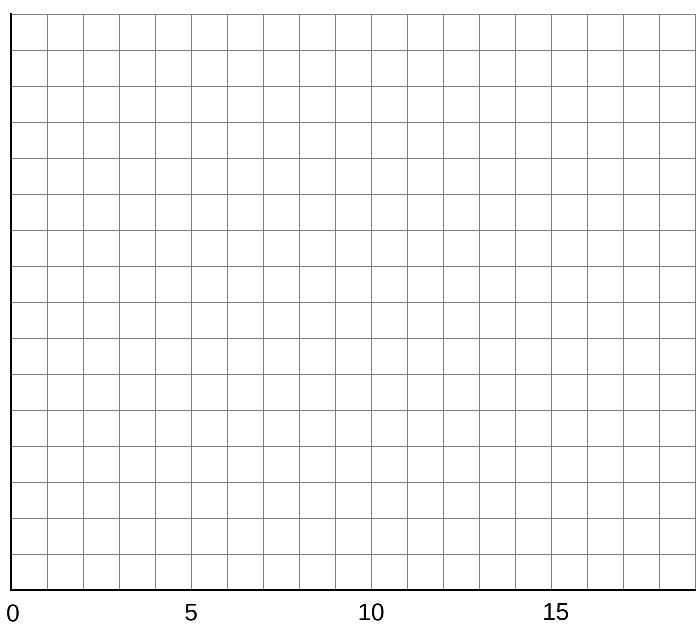

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