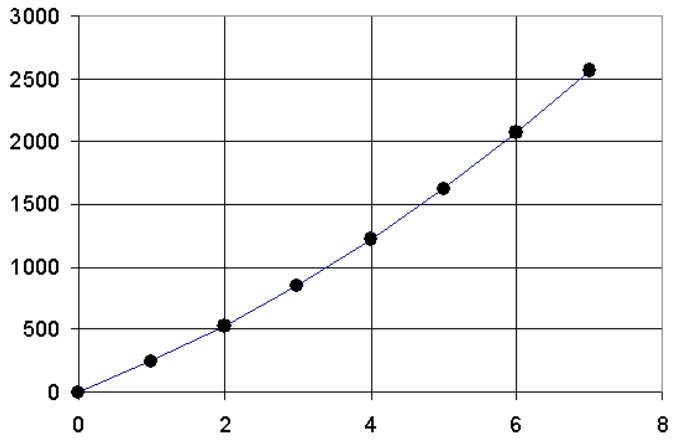
EMM In Class Activites - Quadratic Growth

Problem 1: Finding Quadratic Growth in Data : The tape recorder problem.
This project concerns a tape recorder with a numerical counter. The idea is to model the relationship between the counter and the length of time of a selection on the tape. The data in Table 1 are reprinted from the article *Kinematics of Tape Recording* by J. P. McKelvey, American Journal of Physics, January, 1981.[1] Although modern technology no longer uses magnetic tape recorders, the mathematics involved in this problem can be applied to industrial settings where some product is rolled onto a spool. Examples include textiles (such as carpeting) and paper goods (such as paper towels).

The question McKelvey studied was how a mechanical counter for turns of the takeup reel varies over time. Using a stop watch, he watched the counter and every 100 units wrote down the stop watch reading. His results are shown in the first two columns of the table below. Note that the actual counter number is 100 times the position number. A graph of the data is shown beside the table. To look for patterns, fill in the columns for the first and second differences.

Position Number n	Time (seconds) t_n	First Diffs	Second Diffs
0	0		
1	242		
2	527		
3	854		
4	1220		
5	1626		
6	2072		
7	2558		



[1] In the original table times were recorded in minutes and seconds. They have all been converted to seconds here.

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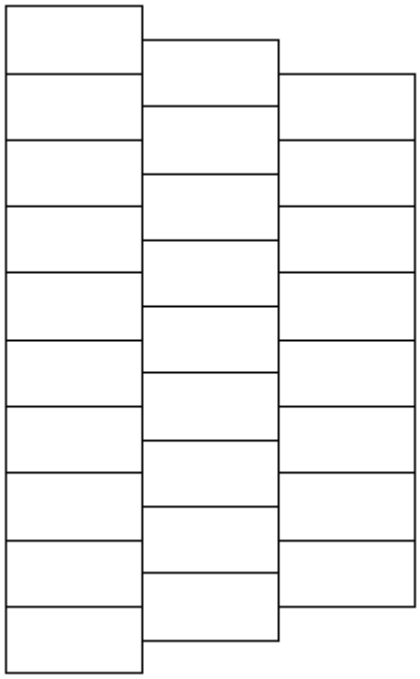
Problem 2: Quadratic Growth that Arises Structurally : The handshake problem.
In the kingdom of Kalmania it is the custom that in any gathering with the king, each citizen must shake every other citizen's hand, but the king must shake every other person's hand 3 times. Suppose the king is in a gathering with 400 subjects. How many handshakes in all will occur?



Instead of considering only the question asked, with 400 subjects, we can more generally study how the number of handshakes varies with the number of subjects. With only 1 subject it is easy to count the number of handshakes – 3. In fact, with 0 subjects we can see that there will be 0 handshakes. Enter 0 and 3 in the first and second boxes of the first column at left. Then figure out how many handshakes will occur if there are 2 subjects or 3 subjects, and enter those in the next two boxes in the column. Finding first and second differences you will see a definite pattern.

We can see that this pattern holds up, as follows. Suppose 200 subjects are at a gathering, and have all shaken hands the required number of times. Now the 201st subject arrives. How many additional handshakes does that require? _____.

If we write H_{200} for the number of handshakes with 200 subjects, and H_{201} for the number of handshakes with 201 subjects, your calculation shows that $H_{201} = H_{200} +$ _____.
Now using the same logic, complete this line: $H_{n+1} = H_n +$ _____.

You should have found a difference equation that is in the standard form for quadratic growth. Now you can write down the functional equation as well:
$H_n =$ _____.

Check whether the functional equation gives the same values for H_0, H_1, H_2, and H_3. That you got before. Then use it to find H_{400}. That tells how many handshakes must occur when 400 subjects are present.

NOTE: this problem is stated in terms of handshakes for conceptual simplicity, although no one seriously needs to count handshakes. However, there are many genuine problems connected with networks that have similar characteristics. These can be networks of roads, delivery routes, internet connections, etc. As a general rule, questions about the interconnectivity or capacity of networks are often modeled using quadratic growth models. As in the handshake problem, the applicability of quadratic growth arises because of the structure of the context, as opposed to merely being observed as a pattern in the data.

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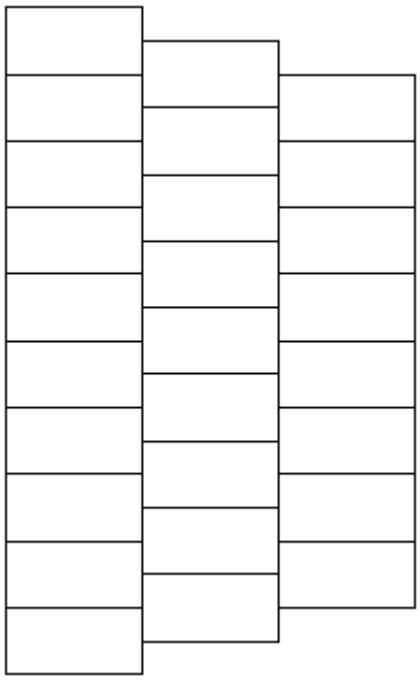
Problem 3: Quadratic Growth from Running Totals of an Arithmetic Growth Model: The epidemic problem.

In a small city in the Kalmanian Mountains, an outbreak of mathitis occurs. When the health service figured out what was going on, there were already 29 confirmed cases of mathitis. The next day there were 14 new cases, the day after there were 18 new cases, the third day there were 22 new cases. Assuming this pattern continues, how long will it be before every one of the city's 73,000 residents has mathitis?



As before, you can work out the first several days worth of data directly from the given information. Let us label things so that there were 29 cases on day 0. Enter a 29 in the top left box of the table. Now compute and enter in the same column the total number of infected residents for day 1, day 2, day 3, and day 4. Next compute the first and second differences. Use the pattern you see to analyze this problem. Find difference and functional equations and then predict when everyone will have mathitis.

NOTE: the given information tells us that the number of *new* cases of mathitis each day follows an arithmetic growth pattern: the number of new cases each day begins 14, 18, 22, and continues in that pattern. The *total* number after *n* days can be obtained by computing a running total based on the new cases each day. (For this problem, we would have to add 29 to that running total, because on the first day there are already 29 cases.) It is often of interest to look at a running total for a model. When the original model is arithmetic growth, the running total will always exhibit quadratic growth.

Elementary Math Models
In class activity: Quadratic Growth Model in Context

A development plan for a city has identified 60,000 developable lots for houses. Over several years the planning commission keeps track of the new houses that are built, obtaining the following approximate figures:

Year	0	1	2	3	4	5
Houses Built	3000	3800	4600	5400	6200	7000
Running Total	3000	6800	11400			

Assuming that the number of houses built each year continues to grow according to the pattern in the table, you are to estimate how many years it will take to develop all 60,000 available lots. As you answer the questions below, remember to write down explicit definitions (in full sentences) for any variables that you use in equations in your model.

Specific Questions

1. What kind of growth do you observe for the number of houses built in each year?
2. Develop an equation for the number of houses built in the *n*th year.
3. Why can you be sure that the running total row of the table shows quadratic growth?
4. Develop a quadratic growth model for the running total row. In particular, find an equation that will predict the running total entry for year *n*.
5. How many years will it take before all 60,000 lots have been developed, according to your model?