Elementary Math Models

Chapter 1 Work Sheet 2

1. For each sequence listed below, fill in the data table showing position numbers (starting with either 0 or 1) as well as terms, and then create a graph. Preferably, your graphs should be produced with technology (such as excel, or a webpage) and printed out. If you do not have access to technology, create your graphs by hand, using graph paper, and plotting the points as accurately as possible. In either case, label the graphs with the appropriate problem numbers, and attach them to the back of this worksheet.

   In addition to making data tables and graphs, for each sequence also describe a pattern using the terminology of *position numbers* and *terms*, and indicate whether the pattern is recursive or direct. For example, in part *a* you could say either

   Each term is three more than the preceding term, a recursive pattern,

   or

   Each term is three times its position number, a direct pattern.

position	term

a. $3, 6, 9, 12, 15, \ldots$

position	term

b. $1, 2, 4, 8, 16, \ldots$

position	term

c. $1, 4, 9, 16, 25, 36, \ldots$

position	term

d. $1, 3, 6, 10, 15, 21, \ldots$

position	term

e. $5, 8, 11, 14, 17, \ldots$

position	term

f. $1, 1, 2, 3, 5, 8, \ldots$

position	term

g. $1, 10, 100, 1000, \ldots$

position	term

h. $5, 55, 555, 5555, 55555, \ldots$

position	term

i. $2, 6, 12, 20, 30, \ldots$

position	term

j. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

position	term

k. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

2. For each part use the described pattern to work out the first 4 terms of a number sequence. Indicate whether or not each pattern is recursive.

   a. Each term of the sequence is found by adding 5 to the preceeding term. The starting term is 3.

   b. Each term of the sequence is found by multiplying the preceeding term by 1/2. The starting term is 80.

   c. Each term of the sequence is found by multiplying the position number by 7 and adding 3 to the result.

   d. Each term of the sequence is found by by multiplying its position number by the preceeding term. The starting term is 1.

3. For each of the tables below, the sequence values were generated using a pattern. The arrows, circles and squares indicate how the pattern works. For each table, work out the next three terms following the given pattern.

n	a_n
1	10
2	10 + 3 = 13
3	13 + 3 = 16
4	16 + 3 = 19
5	
6	
7	

n	b_n
0	13
1	13 − 1 = 12
2	12 − 2 = 10
3	10 − 3 = 7
4	
5	
6	

n	c_n
1	$\frac{1(1+1)}{2} = 1$
2	$\frac{2(2+1)}{2} = 3$
3	$\frac{3(3+1)}{2} = 6$
4	
5	
6	
7	

4. For the tables in the preceding problem, the indicated computational pattern can be expressed algebraically. For example, the lines of the first table are represented by the equations

$$\begin{aligned} a_1 &= 10 \\ a_2 &= a_1 + 3 \\ a_3 &= a_2 + 3 \\ a_4 &= a_3 + 3 \end{aligned}$$

Find equations for the lines of the other two tables.

5. Rewrite your answers to the preceding problem using parenthesis notation instead of subscript notation. For example, write $a(0)$ in place of a_0, $a(3)$ in place of a_3, and so on.

6. For each part below, several equations are given using subscript notation. For each one, write the next two equations and also the 10th equation for the same pattern with subscript notation. Then create a table of values like the ones in problem 4, showing how the pattern is used to fill the table. Which of the patterns is recursive?

$$a_0 = 17 \qquad b_0 = 0 \cdot 1 + 5 \qquad c_0 = 3$$
$$a_1 = a_0 - 4 \qquad b_1 = 1 \cdot 2 + 5 \qquad c_1 = c_0(c_0 - 1)$$
$$a_2 = a_1 - 4 \qquad b_2 = 2 \cdot 3 + 5 \qquad c_2 = c_1(c_1 - 1)$$
$$a_3 = a_2 - 4 \qquad b_3 = 3 \cdot 4 + 5 \qquad c_3 = c_2(c_2 - 1)$$

7. Rewrite the equations in the preceding problem using parenthesis notation rather than subscript notation. [Hint: the equation $c_1 = c_0(c_0 - 1)$ is written with parenthesis notation like this: $c(1) = c(0)(c(0) - 1)$.]