Elementary Math Models
Chapter 1 Work Sheet 3

1. Some number sequences from a previous work sheet are shown below. For each one, based on a pattern you found earlier, write down several equations using subscript notation, and then rewrite with parenthesis notation. Two different solutions are shown for the sample item, but you only need to provide one solution in completing the parts of this problem.

Sample: 3, 6, 9, 12, 15, ...

Solution 1	
Subscripts	Parentheses
$a_1 = 3$	$a(1) = 3$
$a_2 = a_1 + 3$	$a(2) = a(1) + 3$
$a_3 = a_2 + 3$	$a(3) = a(2) + 3$
$a_4 = a_3 + 3$	$a(4) = a(3) + 3$

Solution 2	
Subscripts	Parentheses
$a_1 = 3 \cdot 1$	$a(1) = 3 \cdot 1$
$a_2 = 3 \cdot 2$	$a(2) = 3 \cdot 2$
$a_3 = 3 \cdot 3$	$a(3) = 3 \cdot 3$
$a_4 = 3 \cdot 4$	$a(4) = 3 \cdot 4$

a. $1, 2, 4, 8, 16, ...$

b. $1, 4, 9, 16, 25, 36, ...$

c. $5, 8, 11, 14, 17, ...$

d. $1, 1, 2, 3, 5, 8, ...$

e. $0, 2, 6, 12, 20, 30, ...$

f. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, ...$

2. For each part a description is given for a pattern. Express the first four terms as well as the tenth term of the pattern using subscript notation, and repeat using parenthesis notation. Indicate whether each pattern is recursive.

Sample: Each term of the sequence is found by adding 5 to the preceeding term. The starting term is $a_0 = 3$.

$$
\text{Subscript Solution:} \quad
\begin{aligned}
a_0 &= 3 \\
a_1 &= 5 + a_0 \\
a_2 &= 5 + a_1 \\
a_3 &= 5 + a_2 \\
a_9 &= 5 + a_8 \\
&\text{recursive}
\end{aligned}
\qquad
\text{Parenthesis Solution:} \quad
\begin{aligned}
a(0) &= 3 \\
a(1) &= 5 + a(0) \\
a(2) &= 5 + a(1) \\
a(3) &= 5 + a(2) \\
a(9) &= 5 + a(8) \\
&\text{recursive}
\end{aligned}
$$

a. Each term of the sequence is found by multiplying the preceeding term by $1/2$. The starting term is $a_0 = 80$.

b. Each term of the sequence is found by multiplying the position number by 7 and adding 3 to the result.

c. Each term of the sequence is found by by multiplying its position number by the preceeding term. The starting term is $a_1 = 1$.

In the next series of problems you will work with variable subscript difference equations and graphs. In some of the patterns, you will find that the numbers get quite *messy*, so be sure you have a calculator handy to do the computations. You are also asked to explore each problems using an Excel spreadsheet as discussed in class. Graphs from the computer should be printed, cut out and attached at the bottom of each page. Alternatively, print and label all your graphs and attach them to the end of the packet.

3. A number sequence begins with $a_0 = 6$ and then follows a pattern as illustrated by the following equations:

$$
\begin{aligned}
a_1 &= a_0 + 1.5 \\
a_2 &= a_1 + 1.5 \\
a_3 &= a_2 + 1.5 \\
a_4 &= a_3 + 1.5
\end{aligned}
$$

a. Write a difference equation for this pattern using a variable subscript. (Hint: it should begin $a_{n+1} =$)

b. Find the numerical values of the terms a_1, a_2, a_3, a_4.

c. Write a version of the difference equation using parenthesis notation

d. Create an excel spreadsheet for this sequence. Use your difference equation to generate the terms of the pattern and then compare the data values shown by the computer with what you found above. Use Excel to create a graph of your data.

4. a. Write a difference equation with variable subscripts for a number sequence beginning with $a_0 = 6$ and then given by the following pattern of equations:

$$
\begin{aligned}
a_1 &= a_0 + 1.5 - 0 \\
a_2 &= a_1 + 1.5 - 1 \\
a_3 &= a_2 + 1.5 - 2 \\
a_4 &= a_3 + 1.5 - 3
\end{aligned}
$$

b. Find the numerical values of the terms a_1, a_2, a_3, a_4.

c. Write a version of the difference equation using parenthesis notation

d. Create an excel spreadsheet for this sequence. Use your difference equation to generate the terms of the pattern and then compare the data values shown by the computer with what you found above. Use Excel to create a graph of your data.

5. a. Write a difference equation with variable subscripts for a number sequence beginning with $a_0 = 25$ and then given by the following pattern of equations:

$$
\begin{aligned}
a_1 &= a_0 \cdot .8 \\
a_2 &= a_1 \cdot .8 \\
a_3 &= a_2 \cdot .8 \\
a_4 &= a_3 \cdot .8
\end{aligned}
$$

b. Find the numerical values of the terms a_1, a_2, a_3, a_4.

c. Write a version of the difference equation using parenthesis notation

d. Create an excel spreadsheet for this sequence. Use your difference equation to generate the terms of the pattern and then compare the data values shown by the computer with what you found above. Use Excel to create a graph of your data.

6. a. Write a difference equation with variable subscripts for a number sequence beginning with $a_0 = 100$ and then given by the following pattern of equations:

$$
\begin{aligned}
a_1 &= .6a_0 + 50 \\
a_2 &= .6a_1 + 50 \\
a_3 &= .6a_2 + 50 \\
a_4 &= .6a_3 + 50
\end{aligned}
$$

b. Find the numerical values of the terms a_1, a_2, a_3, a_4.

c. Write a version of the difference equation using parenthesis notation

d. Create an excel spreadsheet for this sequence. Use your difference equation to generate the terms of the pattern and then compare the data values shown by the computer with what you found above. Use Excel to create a graph of your data.

7. a. Write a difference equation with variable subscripts for a number sequence beginning with $a_0 = 10$ and then given by the following pattern of equations:

$$
\begin{aligned}
a_1 &= .025(100 - a_0)a_0 \\
a_2 &= .025(100 - a_1)a_1 \\
a_3 &= .025(100 - a_2)a_2 \\
a_4 &= .025(100 - a_3)a_3
\end{aligned}
$$

b. Find the numerical values of the terms a_1, a_2, a_3, a_4.

c. Write a version of the difference equation using parenthesis notation

d. Create an excel spreadsheet for this sequence. Use your difference equation to generate the terms of the pattern and then compare the data values shown by the computer with what you found above. Use Excel to create a graph of your data.