Extra Problems 6.1

Let A and B be nonempty sets, and consider a function $f : A \to B$. For any $C \subseteq A$ we define the set

$$f(C) = \{ f(x) \mid x \in C \}.$$

This is called the *Image* of the set C. It can also be written

$$f(C) = \{ y \in B \mid y = f(x) \text{ for some } x \in C\} = \{ y \in B \mid (\exists x \in C)(f(x) = y)\} .$$

Similarly, for any $D \subseteq B$ we define the set

$$f^{-1}(D) = \{ x \in A \mid f(x) \in D \},$$

called the *inverse image of D under f*, or the *pre-image of D under f*. These definitions are referred to in the following problems. For all problems, let A and B be nonempty sets, and consider a function $f : A \to B$.

6.1.x1 Show that $f^{-1}(B) = A$. Is $f(A) = B$?

6.1.x2 Suppose that G and H are subsets of A. Show: If $G \subseteq H$ then $f(G) \subseteq f(H)$.

6.1.x3 Suppose that J and K are subsets of B. Show: If $J \subseteq K$ then $f^{-1}(J) \subseteq f^{-1}(K)$.

6.1.x4 If G and H are subsets of A show that $f(G \cap H) \subseteq f(G) \cap f(H)$. Must $f(G \cap H) = f(G) \cap f(H)$?

6.1.x5 If J and K are subsets of B show that $f^{-1}(J \cap K) \subseteq f^{-1}(J) \cap f^{-1}(K)$. Must $f^{-1}(J \cap K) = f^{-1}(J) \cap f^{-1}(K)$?