# Extra Problem 6.3

Let $A$ and $B$ be nonempty sets, and consider a function $f: A \to B$. For any $C \subseteq A$ we define the set

$$f(C) = \{ f(x) \mid x \in C \}.$$

This is called the *Image* of the set $C$. It can also be written

$$f(C) = \{ y \in B \mid y = f(x) \text{ for some } x \in C\} = \{ y \in B \mid (\exists x \in C)( f(x) = y)\} .$$

Similarly, for any $D \subseteq B$ we define the set

$$f^{-1}(D) = \{ x \in A \mid f(x) \in D \},$$

called the *inverse image of D under f*, or the *pre-image of D under f*. These definitions are referred to in the following problems. For all problems, let $A$ and $B$ be nonempty sets, and consider a function $f: A \to B$.

6.3.x1 Prove the following proposition: If $G$ and $H$ are subsets of $A$ then $f(G \cup H) = f(G) \cup f(H)$.

6.3.x2 If $f$ is an injection (ie. $f$ is a one to one function), prove the following proposition: If $G$ and $H$ are subsets of $A$ then $f(G \cap H) = f(G) \cap f(H)$.

6.3.x3 Prove the following proposition: If $f$ is a surjection, and if $J$ is a subset of $B$ then $f(f^{-1}(J)) = J$.

6.3.x4 Prove the following proposition: If $f$ is an injection, and if $G$ is a subset of $A$ then $f^{-1}(f(G)) = G$.