

Extra Problem 7.2

7.2.x1 Let R be an equivalence relation on \mathbb{Z} , represented by the symbol \equiv . That is, write $a \equiv b$ whenever $(a, b) \in R$. Also, define sets $H = \{n \in \mathbb{Z} \mid n \equiv 29\}$ and $K = \{n \in \mathbb{Z} \mid n \equiv 42\}$. Prove: If $H \cap K$ is nonempty iff $29 \equiv 42$.

7.2.x2 Let A be a nonempty set. Let $R = A \times A$. This is a relation on A . Is it an equivalence relation? Prove it is or prove that it is not.

7.2.x3 Let R be an equivalence relation on \mathbb{Z} , represented by the symbol \equiv . That is, write $a \equiv b$ whenever $(a, b) \in R$. Also, define sets $H = \{n \in \mathbb{Z} \mid n \equiv 29\}$ and $K = \{n \in \mathbb{Z} \mid n \equiv 42\}$. Prove: If $H \cap K$ is nonempty then $H = K$.

7.2.x4 Suppose that R is a relation on the nonempty set A , and suppose that R is symmetric and transitive. Furthermore, suppose that $\text{dom}(R) = A$. Prove that R is an equivalence relation.