Extra Problems 9.2

9.2.x1 Let $n \in \mathbb{N}$ and let $A_1, A_2, A_3, \cdots, A_n$ be countably infinite sets. Prove that the Cartesian product $A_1 \times A_2 \times A_3 \times \cdots \times A_n$ is countably infinite. (Hint: use exercise 9.10 and induction.)

9.2.x2 Let $A_1, A_2, A_3, \cdots,$ be pairwise-disjoint countably infinite sets. Prove that the union $A_1 \cup A_2 \cup A_3 \cup \cdots$ is countably infinite. (Hint: compare this union to an appropriate Cartesian product of countable sets.)

Comments: In problem x1 there are only finitely many sets being combined in a Cartesian product, so induction is valid. For problem x2 an induction argument won't work – that can only show that any union of a *finite* number of sets is countable, and here you are asked to show that the union of an infinite number of sets is countable. Also, problem x1 is not correct with a Cartesian product of an infinite number of sets. For example, if $A = \{0, 1\}$, then the Cartesian product of a countably infinite number of copies of A is essentially the set of sequences of zeros and ones. By an infinite version of the dodgeball game, you can see that the set of such sequences is actually uncountable.