Sample Math Writing Problems

2.1 number 5

If a ball is thrown into the air with a velocity of 40 ft/sec, its height after t seconds is given by $y = 40t - 16t^2$.

(a) Find the average velocity for the time beginning when $t = 2$ and lasting

(i) .5 seconds	(ii) .1 second
(iii) .05 seconds	(iv) .01 seconds

Format: write in complete sentences with a wide left margin, but otherwise normal paragraph formatting. Math symbols may be included in line as part of the running text, or displayed centered on a separate line. These are the *only* generally accepted options. If there are several equations, you can number them and refer to them by number.

Solution. For part (i), we want to find the average velocity between time 2 and time 2.5. At time 2 the height is given by $y_1 = 40*2 - 16*2^2 = 80 - 64 = 16$. At time 2.5 the height is given by $y_2 = 40*2.5 - 16*2.5^2 = 100 - 100 = 0$. So the ball falls 16 feet in one half second between the two times. That means the average velocity would be

$$\frac{-16 \text{ ft}}{.5 \sec} = -32 \frac{\text{ft}}{\sec}$$

For parts (ii) through (iv) we use a similar method. For each case, we compute

$$y_2 = 40t - 16t^2$$

where t is given as 2.1 in (ii), 2.05 in (iii), and 2.01 in (iv). Once we know y_2, we can determine the distance traveled as $y_2 - 16$, and then divide that by the length of time traveled to obtain the average velocity. The results are shown in the following table:

Feel free to include tables and graphs with your solution. The written part of the answer should make reference to this material, and describe it in some fashion.

t_1	y_1	t_2	y_2	Distance $= y_2 - y_1$	Time $= t_2 - t_1$	Average Velocity
2	16	2.1	13.44	-2.56	.1	-25.6
2	16	2.05	14.76	-1.24	.05	-24.8
2	16	2.01	15.7584	-2416	.01	-24.16

(b) Find the instantaneous velocity when $t = 2$.

Solution. The instantaneous velocity is approximated by the average velocities over very short time intervals starting at time 2, and the shorter the time interval, the better the approximation. In the table, we observe that the entries in the last column are successively better approximations to the instantaneous velocity. Looking at the pattern of values, it would be reasonable to conjecture that -24 feet per second might be the correct value of instantaneous velocity. Of course, the three data values in the table do not prove that this figure is correct, and we could try some additional average velocities over even shorter time periods to see whether -24 still seems a reasonable figure. But there is no assurance at this point that the figure of -24 is absolutely correct.

2.2 number 1.
Explain in your own words what is meant by the equation

$$\lim_{x \to 2} f(x) = 5$$

Is it possible for this statement to be true and yet $f(2) = 3$?

Notice that this answer has no calculation in it. It explains what is meant by a certain concept, and that doesn't necessarily require algebra or computation.

Solution. The limit equation means that for values of x near (but not equal) to 2, the function values $f(x)$ are near 5. In addition, the closer the x values are taken to 2, the closer the resulting $f(x)$ values are supposed to get to 5. Thus, in a table of values, choosing x values closer and closer to 2, both greater and less than 2, will lead to a pattern of $f(x)$ values approaching 5. There is also a graphical interpretation to the limit equation. If we imagine a graph of the equation $y = f(x)$ from which the point at $x = 2$ has been excluded, there should be a curve with a single missing point at (2,5). This point should match up with the graph of the curve on both the right and the left. And this appearance should persist no matter how much we magnify the curve right around the area of the point (2,5).

It is possible for the limit to equal 5 even though $f(2) = 3$. We saw an example in class that the for the function $f(x) = \text{int}(1 - x^2)$, the limit as x approaches 0 is 0, but $f(0) = 1$. In fact, we can create the related function

$$f(x) = 5 - 2\ \text{int}(1 - (x-2)^2)$$

and for this function, the limit as x approaches 2 is 5, but $f(2) = 3$.