Day 7: Tuesday, 2/7/2017

Collect 4.1 Quiz Problem; 4.2, 4.3 regular homework. Delay 4.3 if students wish. Take questions about these assignments if there are any.

Continue with the example from previous lecture, demonstrate all the steps to show there is a limit and calculate its value. Include discussion of order preserving and order reversing functions.

New material: 5.1 Sets

1. Overview: set notation and operations are everywhere in mathematics. Understanding this material and the basics of proving things with and about sets are crucial for further math study.
2. The basics: concept of a set as a collection. Roster, description, and set builder specification of sets. Finite and infinite sets, conceptually. The empty set.
3. Set operations
   a. Union
   b. Intersection
   c. Complement (relative to a universal set U).
   d. Relative compliment aka set subtraction or set difference
4. Venn Diagrams
   a. Graphical representation of set relationships
   b. Like graphs in calculus, these diagrams can help you understand relationships, but are not considered adequate to prove results in most cases
   c. There are diagrammatic interpretations for unions, intersections, complements, and combinations of these.
   d. Example: Venn diagram shows that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
5. Subsets and set equality
   a. $A = B$ iff they have identical elements. That means $(\forall x \in U)((x \in A) \leftrightarrow (x \in B))$.
   b. A is a subset of B when every element of a is also an element of B. We also say that B contains A to mean the same thing as A is a subset of B.
   c. Logically, A is a subset of B when $(\forall x \in U)((x \in A) \rightarrow (x \in B))$. Note that if $A = B$ then also A is a subset of B. So *subset of* allows for the possibility of equality.
   d. Notation is $A \subseteq B$, to remind us of the possibility of equality. We also write $B \supseteq A$ (B contains A) to mean the same thing as $A \subseteq B$ (A is contained in B).
   e. Note that $A = B$ iff $A \subseteq B$ and $B \subseteq A$.
   f. If A is a subset of B but is not equal to B, then A is contained in B but does not include every element of B. In this case we say that A is a proper subset of B and write $A \subset B$.
   g. These notations are not universally accepted. In some works the symbol $A \subset B$ means A is a subset of B (not necessarily proper) and $A \subsetneq B$ means A is a proper subset of B.

6. Subset examples
   a. Is $\{x \in \mathbb{R} \mid 2x^3 + x^2 - 2x - 6 = 0\} \subseteq \mathbb{Q}$?
   b. Is $\{x \in \mathbb{R} \mid 2x^3 + x^2 - 2x - 6 = 0\} \subseteq \{x \in \mathbb{R} \mid (x^2 - 7)(2x^3 + x^2 - 2x - 6) = 0\}$?
   c. Is {continuous functions from $\mathbb{R}$ to $\mathbb{R}$} $\subseteq$ {differentiable functions from $\mathbb{R}$ to $\mathbb{R}$}? What if we replace $\subseteq$ with $\supseteq$? With $\supset$?
7. Empty set properties. For every set B …
   a. … $\emptyset \subseteq B \subseteq B$.
   b. … $\emptyset \cup B = B$.
   c. … $\emptyset \cap B = \emptyset$.
8. Power Set of a Set
   a. Definition: $\wp(A)$ is the set of subsets of A.
   b. Example: $A = \{1, 2, 3\}$
   c. Don't confuse $\in$ with $\subseteq$. $\{1,2\} \subseteq A$ but $\{1,2\} \in \wp(A)$
   d. Theorem: If A is a set with n elements, then A has 2^n subsets (including $\emptyset$ and A itself). In other words, $\wp(A)$ has 2^n elements.
9. Cardinality
   a. For a finite set this is the number of elements. We denote it card(A). Some works write $|A|$ for the cardinality of the set A.
   b. In general, without knowing whether sets are finite or infinite, we define two sets to have equal cardinality iff there is a one to one correspondence between the two sets.
   c. Set theoretic definition of infinite: The set A is infinite if and only if there is a proper subset $B \subset A$ such that card(B) = card(A).
   d. Two infinite sets can have different cardinalities. For example card($\mathbb{R}$) $\neq$ card($\mathbb{Q}$).
10. Section ends with review of some of the familiar standard number systems: natural numbers, integers, rationals, irrationals, reals, and complex numbers.

End of Day