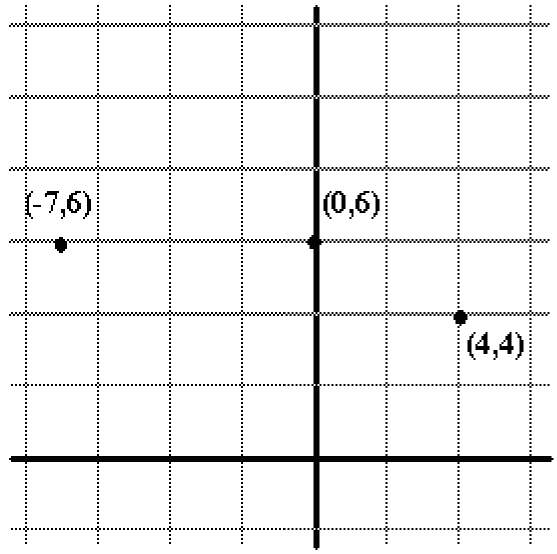
Linear Algebra Worksheet 2: Examples of Systems of Equations

Find a Parabola



A parabola with a vertical axis of symmetry represents the graph of a quadratic function. Does such a parabola pass through the points (-7,6), (0,6) and (4,4)?

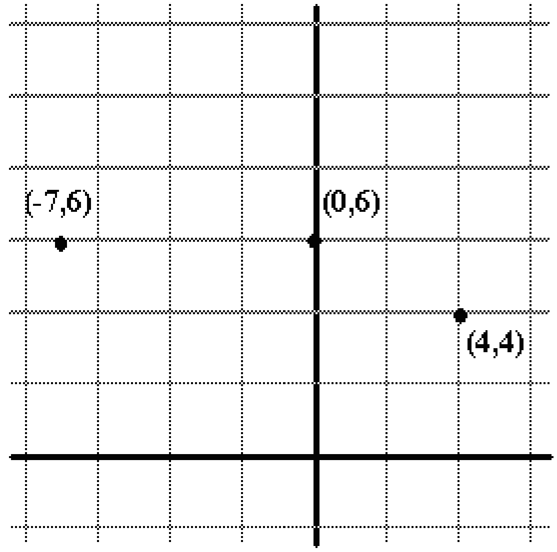
If so, the equation will be of the form

$$y = ax^2 + bx + c$$

Here, the coefficients *a*, *b*, and *c* are unknowns. What equations must you solve to find these unknowns?

Find a Circle

Is there a circle which passes through the points (-7,6), (0,6) and (4,4)?

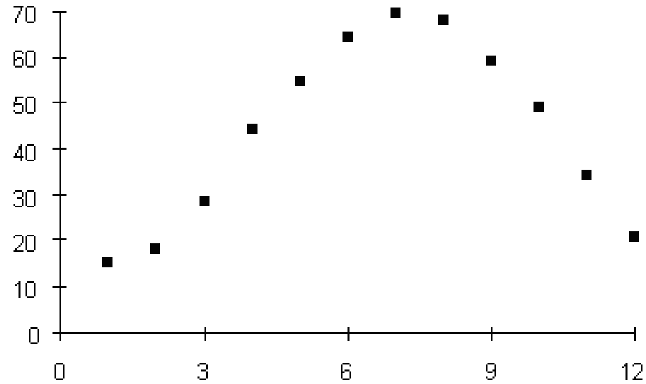


The center and radius of the circle are unknowns. If the center is at (h, k) and the radius is r, the equation of the circle will be

$$(x - h)^2 + (y - k)^2 = r^2$$

What equations must you solve to find h, k, and r?

Find a shifted sin curve.



The graph at right shows the average temperatures by month for Green Bay, Wisconsin, from 1980. The data are shown in the table below. Because the seasons repeat each year, it is reasonable to assume that these average temperatures will fit on some sort of sine shaped curve. Can you find the right equation for that curve?

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
15.4	18.0	28.6	43.8	54.5	64.5	69.2	67.7	58.9	49.2	34.1	20.9

It makes sense to assume that the cycle of repetition is 12, so the functions we are interested in are $\sin(2\pi t/12)$ and $\cos(2\pi t/12)$. Then the general equation for a sine shaped curve will be

$$f(t) = a + b \sin(2\pi t/12) + c \cos(2\pi t/12)$$

Here, *a, b,* and *c* are constants that we want to find. For each data point, we get one equation relating these unknowns. For example, using the first month (January) we see from the table that $f(1) = 15.4$ so by substitution we arrive at:

$$a + b \sin(2\pi/12) + c \cos(2\pi/12) = 15.4$$

Simplifying, $2\pi/12 = \pi/6$, and if you recall your trig, the sine and cosine of $\pi/6$ are 1/2 and $\left(\sqrt{3}\right)/2$, respectively. Using all the data in the table, find the 12 equations that must be satisfied if the curve is to pass through all the data points.

Find a Linear Relation

The table below shows nutritional information for a number of items on the menu at McDonalds. It seems reasonable that there should be some relationship among the different columns. For example, we might expect that if you know the how much protein, carbohydrate, and fat are in an item, that you could use that information to figure out how many calories are in the item. That would mean that there is a function $K(p,c,f)$ that will convert protein (p), carbohydrate (c), and fat (f) into kcalories. The simplest sort of function to consider is LINEAR:

$$K(p,c,f) = rp + sc + tf$$

where r, s, and t are constants that must be determined based on the data. Another way to say the same thing is this: we want to find constants r, s, and t so that when you add r times the protein column to s times the carbo column and t times the fat column that gives you the kcal column. Which ever way you look at it, the unknowns r, s, and t that we want must satisfy a whole series of equations -- one for each line of the table. For example, the equation for the first line of the table would be

$$19r + 31s + 15t = 327.$$

Write down the first 10 of the full set of equations for r, s, and t.

	protein	carbo	fat	kcal	chol
eggmcmuffin	19	31	15	327	229
englishmuff	5	30	5	186	13
hotcakes	8	94	10	500	47
sausage	9	0	19	206	43
scrambleggs	13	3	13	180	349
hashbrowns	2	14	7	125	7
bigmac	26	41	33	563	86
cheeseburger	15	30	14	307	37
hamburger	12	30	10	255	25
quarterpound	24	33	22	424	67
quartercheese	30	32	31	524	96
filetofish	14	37	25	432	47
regularfries	3	26	12	220	9
applepie	2	29	14	253	12
cherrypie	2	32	14	260	13
cookies	4	49	11	308	10
chocoshake	10	66	9	383	30
strawshake	9	62	9	362	32
vanilshake	9	60	8	352	31
fudgesunda	7	46	11	310	18
caralsunda	7	53	10	328	26
strawsunda	7	46	9	289	20

Compartment model

Here is a very simple example of a compartment model: Suppose that a population of penguins nests on two islands, designated East and West. Each year some of the penguins who had been living on East island move to West island, and vice versa, and some of the penguins stay where they were. If 20% of the East Island penguins move to West Island, then 80% stay put. Similarly, if 30% of West Island penguins move to East Island, then 70% stay put. Based on these assumptions, we can track how the populations vary over time. If there are 1000 penguins on East Island and 2000 on West Island this year, then next year on East Island there will be (.80)1000 + .30(2000) penguins – these are the 80% of East Islanders who stayed on East and the 30% of West Islanders who moved. Using these assumptions, how many penguins will be on each island next year?

Using variables, if e and w stand for how many East and West Island penguins there are this year, and if e_1 and w_1 are the East and West island populations a year later, find equations for e_1 and w_1 as functions of e and w.

Working with this model we can ask, if the populations this year are 1000 and 2000, what will they be in 5 years or 10 years? Or, we can reverse the question: if the populations are 1000 and 2000 today, what does the model say they were 10 years ago? These are all questions about linear equations.

Compartment models can be applied in many situations, with compartments representing geographical areas, demographic categories (based on age, income, education, health, political affilition or other attributes), actual compartments of chemical reactants, etc. The number of compartments dictates how many variables will occur in the equations of the system.