Elementay Math Models

Worksheet: Number Patterns 2

1. For each of the tables below, the sequence values were generated using a pattern. The arrows, circles and squares indicate how the pattern works. Use subscript notation to express that pattern for several of the terms of the sequence. The solution is shown for the first pattern.

position	term
0	10
1	10 +3 = 13
2	13 +3 = 16
3	16 +3 = 19

position	term
0	13
1	13 - 1 = 12
2	12 - 2 = 10
3	10 - 3 = 7

position	term
0	$\frac{0(0+1)}{2} = 0$
1	$\frac{1(1+1)}{2} = 1$
2	$\frac{2(2+1)}{2} = 3$
3	$\frac{3(3+1)}{2} = 6$

$a_1 = a_0 + 3$ $\qquad$ $b_1 =$ ____________ $\qquad$ $c_1 =$ ____________

$a_2 = a_1 + 3$ $\qquad$ $b_2 =$ ____________ $\qquad$ $c_2 =$ ____________

$a_3 = a_2 + 3$ $\qquad$ $b_3 =$ ____________ $\qquad$ $c_3 =$ ____________

2. For each part below, several equations are given using subscript notation. For each one, write the next two lines with subscript notation for the same pattern. Then create a table of values like the ones in problem 1, showing how the pattern is used to fill the table, and identify which of these examples feature a recursive pattern.

$a_0 = 17$
$a_1 = a_0 - 4$
$a_2 = a_1 - 4$
$a_3 = a_2 - 4$

$b_0 = 0 \cdot 1 + 5$
$b_1 = 1 \cdot 2 + 5$
$b_2 = 2 \cdot 3 + 5$
$b_3 = 3 \cdot 4 + 5$

$c_0 = 3$
$c_1 = c_0(c_0 - 1)$
$c_2 = c_1(c_1 - 1)$
$c_3 = c_2(c_2 - 1)$

3. For each part of problem 2, write a verbal description of the pattern. The answer for part a is:

Any term minus 4 produces the next term.

Use this style to describe the patterns. Note that your description may refer to both terms and position numbers.

4. Several number patterns from the previous work sheet are shown below. For each one, write down several examples of the pattern using subscript notation. Two solutions are shown for the first example, but you need only one solution for each part.

a. 3, 6, 9, 12, 15, ...

$$\begin{aligned} a_1 &= 3 \\ a_2 &= a_1 + 3 \\ a_3 &= a_2 + 3 \\ a_4 &= a_3 + 3 \end{aligned}$$

OR

$$\begin{aligned} a_1 &= 3 \cdot 1 \\ a_2 &= 3 \cdot 2 \\ a_3 &= 3 \cdot 3 \end{aligned}$$

b. $1, 2, 4, 8, 16, ...$

c. $1, 4, 9, 16, 25, 36, ...$

d. $5, 8, 11, 14, 17, ...$

e. $1, 1, 2, 3, 5, 8, ...$

f. $1, 10, 100, 1000, ...$

g. $5, 55, 555, 5555, 55555, ...$

h. $1, 2, 6, 12, 20, 30, ...$

i. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, ...$

j. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, ...$

5. For each part use the described pattern to write several equations using subscript notation. Indicate whether each pattern is recursive.

a. Any term of the sequence plus 5 produces the next term. The starting term is 3.

Solution:

$$a_0 = 3$$
$$a_0 + 5 = a_1$$
$$a_1 + 5 = a_2$$
$$a_2 + 5 = a_3$$

b. Any term of the sequence tijmes 1/2 produces the next term. The starting term is 80.

c. Each term of the sequence is found by multiplying the position number by 7 and adding 3 to the result.

d. Any term of the sequence times the next position number produces the next term. The starting term is 1.

6. For each pattern in problem 5, translate the pattern into a difference equation with variable subscripts. In making the translation, leave numbers and operations unchanged, but use the following table of substitutions:

Replace this:	with this:
any term	a_n
the next term	a_{n+1}
position number	n
the next position number	$n + 1$
produces	$=$

After you have written your translated equation for each part, rewrite the equation by exchanging the two sides of the =.

Solution for part a:

Original:	Any term of the sequence	plus 5	produces	the next term
Translated:	a_n	$+ 5$	$=$	a_{n+1}
Reversed:	$a_{n+1} = a_n + 5$			