Elementay Math Models
Worksheet: Number Patterns 3

In this series of problems, you will study several examples of number patterns described by difference equations. This will allow you to practice using subscript notation and drawing graphs. In some of the patterns, you will find that the numbers get quite *messy*, so be sure you have a calculator handy to do the computations. You will have to round off these messy numbers in order to graph them. You will explore the same set of problems in the computer lab. The directions for the computer lab follow the series of problems.

1. A number sequence begins with $a_0 = 6$ and then follows a pattern as illustrated by the following equations:

$$
\begin{aligned}
a_1 &= a_0 + 1.5 \\
a_2 &= a_1 + 1.5 \\
a_3 &= a_2 + 1.5 \\
a_4 &= a_3 + 1.5
\end{aligned}
$$

   a. Write a difference equation for this pattern using a variable subscript. (Hint: it should begin $a_{n+1} =$)

   b. Find the numerical values of the terms a_1, a_2, a_3, a_4.

   c. On a separate piece of graph paper, draw a graph of this pattern

2. a. Write a difference equation with variable subscripts for a number sequence beginning with $a_0 = 6$ and then given by the following pattern of equations:

$$
\begin{aligned}
a_1 &= a_0 + 1.5 - 0 \\
a_2 &= a_1 + 1.5 - 1 \\
a_3 &= a_2 + 1.5 - 2 \\
a_4 &= a_3 + 1.5 - 3
\end{aligned}
$$

   b. Find the numerical values of the terms a_1, a_2, a_3, a_4.

   c. On a separate piece of graph paper, draw a graph of this pattern

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3. a. Write a difference equation with variable subscripts for a number sequence beginning with $a_0 = 25$ and then given by the following pattern of equations:

$$
\begin{aligned}
a_1 &= a_0 * .8 \\
a_2 &= a_1 * .8 \\
a_3 &= a_2 * .8 \\
a_4 &= a_3 * .8
\end{aligned}
$$

b. Find the numerical values of the terms a_1, a_2, a_3, a_4.

c. On a separate piece of graph paper, draw a graph of this pattern

4. a. Write a difference equation with variable subscripts for a number sequence beginning with $a_0 = 100$ and then given by the following pattern of equations:

$$
\begin{aligned}
a_1 &= .6a_0 + 50 \\
a_2 &= .6a_1 + 50 \\
a_3 &= .6a_2 + 50 \\
a_4 &= .6a_3 + 50
\end{aligned}
$$

b. Find the numerical values of the terms a_1, a_2, a_3, a_4.

c. On a separate piece of graph paper, draw a graph of this pattern

5. a. Write a difference equation with variable subscripts for a number sequence beginning with $a_0 = 10$ and then given by the following pattern of equations:

$$
\begin{aligned}
a_1 &= .025(100 - a_0)a_0 \\
a_2 &= .025(100 - a_1)a_1 \\
a_3 &= .025(100 - a_2)a_2 \\
a_4 &= .025(100 - a_3)a_3
\end{aligned}
$$

b. Find the numerical values of the terms a_1, a_2, a_3, a_4.

c. On a separate piece of graph paper, draw a graph of this pattern

LAB INSTRUCTIONS

In the computer lab, you will enter each of the difference equations that you developed on the preceding sheet into the computer, and have the computer calculate the terms of the sequence and create a graph. A separate handout will instruct you how to use the computer.

When you enter each difference equation, you will have to put it into a form that is appropriate for the computer. First of all, your difference equation should be in the form

$$a_{n+1} = \text{Something to do with } a_n$$

For example, for problem 1, it should be

$$a_{n+1} = a_n + 1.5$$

Note that an equally valid form of this equation is

$$a_n = a_{n-1} + 1.5$$

That is the form that is needed with the TI82 calculator but that is not the one we need to use with the computer. So be sure your difference equation has a_{n+1} on the left side.

Next, the computer does not allow equations to be entered using subscripts. Instead, a notation with parenthesis is used. In this notation, you type `a(n+1)` instead of a_{n+1} and `a(n)` instead of a_n. Write each of your equations in that form before you begin the lab activities.

Finally, as you work with each difference equation, use the computer to graph it two ways. First create the bar graph, then clear the screen and recompute to create a line graph. The point of this is to see some of the different shapes that the graphs can take, and to compare with the graphs you drew by hand.