Elementay Math Models

Group Project: Quadratic Growth Tape Recorder Model

Introduction

This project concerns a tape recorder with a numerical counter. The idea is to model the relationship between the counter and the length of time of a selection on the tape. The data in Table 1 are reprinted from the article *Kinematics of Tape Recording* by J. P. McKelvey, American Journal of Physics, January, 1981.

Model Context

On many open reel and cassette audio tape recorders, there is a counter that advances as the tape is played. The counter can be used to find a particular part of a recording. For example, if you record 15 songs on a tape, and write down the counter number at the start of each, you can later queue the tape up to the start of a particular song by advancing the tape to the appropriate reading on the counter.

It is tempting to use these counter readings as an indication of how long a song is. However, observation shows that this is not a simple matter. If the counter advances 50 units during a three minute song at the *start* of the tape, that same song would **NOT** take 50 units on the counter at the *end* of the tape.

In studying this situation, McKelvey used a stop watch to see how the playing time for a tape correlated with the counter on his tape recorder. He watched the counter and every 100 units wrote down the stop watch reading. Here is what he found:

Reading Number	Time (min:sec)
0	0:00
1	4:02
2	8:47
3	14:14
4	20:20
5	27:06
6	34:32
7	42:38

Table 1. Counter Readings (in hundreds) and Times

Keep in mind that the data points were taken every 100 units on the counter. That means that Reading Number 3, for example, corresponds to a counter value of 300.

Project Directions

1. Create a new table, changing each time value from minutes and seconds to some number of seconds. For example, 4:02 means 4 minutes and 2 seconds. Since each minute is 60 seconds,

4 minutes would be 240 seconds, so 4:02 becomes 242 seconds. Create your table showing the number of seconds for each data point.

2. Compute the first and second differences for your data. You should find that the second differences are nearly constant.

3. Create a quadratic growth difference equation that approximates your data, as follows. Compute the average of the second differences you found in your data. Use this as the fixed second difference for the model, and so as the parameter e for the difference equation. For the d parameter, use the first of the first difference values you found in the data. Write down the difference equation and figure out the functional equation.

4. Experiment with the demodel.mwt computer module to see if you can get a model that is closer to the actual data. Enter your data in the table you see on the first screen, then go to the next screen and enter your difference equation. Also enter settings that will generate data values from $n = 0$ to $n = 7$. How do these compare with the real data? You can see the graph of the model and the actual data points, and can see the numerical differences between the model values and the original data values. Now make some small change in the d or e parameter and recompute. Do the data values produced by the computer come any closer to the real data? Try a few experiments and see if you can choose d and e to best match the actual data. Whatever difference equation you end up with, also write down the functional equation that goes with it.

5. Use the *Graph Functional Equation* button to verify that your functional equation is the correct one for your difference equation. If it is, the graph of the functional equation should perfectly match the graph you already have for the difference equation.

6. Use the model. Suppose that a song on the tape goes from a counter reading of 225 to 305. How many seconds does that song last? To answer this, use your functional equation to change each counter reading to a number of seconds, and then subtract. Note that 225 on the counter is equivalent to a value of 2.25 for x because the counter values for the real data are in units of hundreds.

   Here is a second application. If you start playing the tape at the beginning, what will the counter show after 20 minutes? Here you know the time (20 minutes = 1200 seconds) and you want to figure out the counter reading.

7. Extra Credit: Find an equation for the counter reading as a function of the time. To do this, you will have to use the functional equation that gives seconds (y) equal to some formula involving the counter number (x), and invert that to get x isolated on one side of the equation. This will require the quadratic formula.