Elementay Math Models

Lab Activity for Chapter 12

In this lab you will explore some different kinds of mixed models involving aspects of both arithmetic and geometric growth. All of the examples you work with will involve variations of this difference equation:

$$a_{n+1} = \frac{3}{4}a_n + 100$$

You will use the diffeq.mwt module to see how models of this type perform. Remember that you must reexpress the difference equation in order to use the module. For example, write the difference equation

$$a_{n+1} = \frac{3}{4}a_n + 100$$

in the form

$$a(n+1) = (3/4) * a(n) + 100$$

1. The medicine dosage model. A patient is given a dose of medication every four hours. It is known that in four hours, the body will remove approximately 1/4 of this drug from the blood stream. Suppose that the patient is initially given 500 units of the medicine, and then an additional 100 units every 4 hours. After the first dose is taken, there are 500 units of the drug in the blood stream. At the end of four hours, 1/4 of that has been removed, so that only 375 units remain. Then another 100 units are taken, boosting the amount of drug in the blood to 475. After another 4 hours, 1/4 of the 475 is removed, leaving 3/4 of 475, or 356.25, then 100 units are added, for a total of 456.25 units of the drug in the blood stream. The general pattern is this: Write down the amount of drug in the blood immediately after each dose is taken, with $a(0)$ being the amount of drug immediately after the starting dose, $a(1)$ the amount immediately after the first repeated dose, and so on. Then the pattern of numbers you get is described by

$$a(n+1) = (3/4) * a(n) + 100; \quad a(0) = 500$$

Use the computer module to examine what happens to this model. Does the amount of drug keep building up, or does it eventually level off? Write a brief answer and an explanation here.

2. You should have found in part 1 that the amount of drug in the blood levels off to about 400 units. In this section you will investigate variations of the model. First, keeping everything else about the problem the same, try some different values for the initial dosage. Remember that the repeated dosage is supposed to stay the same, 100 units. So the difference equation will stay the same. Just the starting point, $a(0)$ is supposed to be changed. Use the computer module to examine what happens $a(0)$ is each of the following values: 100, 200, 300, 400. For each of these values, use the computer to generate the first 20 data points and a graph, and answer these questions: Does the amount of drug in the blood still level off? If so, to what? And how long does it take to level off?

Now suppose that the initial dosage is kept fixed at 500 units, but the repeated dosage is changed. Instead of giving the patient 100 units every four hours, what if 50 units are given? 20 units? 200 units? 500 units? How does that change the difference equation? If the repeated dosage is 50 units, for example, the difference equation will be

$$a(n+1) = (3/4) * a(n) + 50$$

Use the computer to see what happens for this difference equation (using $a(0) = 500$). Then repeat the problem using each of the following values for the repeated dosage: 20, 200, 500. Enter your results in the following table:

Repeated Dosage Amount	Level Off Amount
20	
50	
100	400
200	
500	

Can you see a pattern that relates the size of the repeated dosage with the amount at which the drug eventually levels off? Use this pattern to predict where the model will level off if the repeated dosage is 1000 units, then check your prediction on the computer. Write a paragraph about your findings below.

Based on your findings, what should the patient be given as a repeated dosage if you want the drug to level off to about 760 units?

3. In the preceding variations, the amount of drug removed from the body between doses was always kept at 1/4. For different drugs, or for different lengths of time between doses, this fraction can change. Now you will investigate the effects of changing this parameter.

Observe that if 1/4 of the drug is removed, 3/4 remain, and that the 3/4 shows up in the difference equation

$$a(n+1) = (3/4) * a(n) + \text{repeated dose}$$

For convenience we will express 3/4 in decimal form as .75

$$a(n+1) = .75 * a(n) + \text{repeated dose}$$

If the body removes 20% of the drug between doses, that leaves 80% in the body, so the difference equation becomes

$$a(n+1) = .80 * a(n) + \text{repeated dose}$$

Investigate how that effects the model. Repeat the exercises from before using .80 in place of (3/4). Use your results to complete this table:

Repeated Dosage Amount	Level Off Amount
20	
50	
100	
200	
500	

Write a report on your findings below. Include in the report a description of whether the new model eventually levels off, and if so to what value. Describe how the level value is affected by changing the initial dose. How is it effected by changing the repeated dose. Also describe how the level amount can be predicted based on the repeated dose.

4. Repeat the previous page but this time suppose that the body removes 40% of the drug between doses. After you complete your investigation, see if you can find a pattern connecting the percentage of drug removed from the body between doses (that is, the 25%, 20%, and 40% of the examples), the repeated dose amount, and the level amount. It may help to complete the table below. For the last line of the table, fill it in based on a pattern in the table.

Percent of Drug Removed Between Doses	Rule for Finding the Level Amount
25	Multiply Repeated Dose $\times$ 4
20	
40	
10	

Use your entry from the last line of the table to predict what the level amount will be with repeated doses of 50 units assuming that the body removes only 10% of the drug between doses. Then use the computer module to see if your prediction is correct. Remember that the difference equation will be

$$a(n+1) = (.90) * a(n) + 50$$

Write up your conclusions below.