Elementay Math Models

**Lab Activity for Chapter 7**

This laboratory period will be spent working with a few of the ideas from Chapter 7 on polynomials and rational functions. Take notes on the activities as you go along, either using a separate sheet of paper, or using the computer report making module.

## Getting Started

During this lab you will use 3 or 4 of the Mathwright modules. You will need to use the Mathwright Player program to open *graph.mwt* and *1ovrx.mwt*. If you want to use the report module to keep your lab notes, open *report.mwt* as well. Note that you will have several different modules open, but will only see one at a time. Under the word `Window` at the top of the screen there is a menu that can be used to view the different modules. All open modules are listed at the bottom of the menu. Click the one you want to view.

## Graphical and Numerical Methods for Solving Polynomial Equations

In this part of the lab, you will use graphical and numerical methods to find solutions or roots to polynomial equations. For quadratic polynomials, it is always possible to use the quadratic formula. For polynomials of degree 3 or more, this is not possible.[1] The graphical and numerical approach can be used for any kind of function.

Use the `Window` menu to view the *graph.mwt* module. You should click the button for 2–D Graphs. You will enter a function in the white space at the top of the screen and look at the graph. Using *zoom* and *get coordinates* you will find specific points on the graph with high accuracy. Then these results can be refined using a numerical approach. At the bottom of the screen there is a place to enter a numerical value for $x$ and have the $y$ computed.

Here is the first equation:

$$x^4 + 5x^3 + 5x^2 - 5x - 5 = 0$$

Notice that there is only one variable here. If you replace it with a number, you will end up with a single number on each side of the equal sign. For most choices of $x$ the result will be a false statement. For example, if you replace $x$ with 0, the result will be $-5 = 0$, which is definitely false. The point is to find the values of $x$ that make the equation true.

To apply the graphical method in this case, we introduce an additional variable, $y$ and look at the following related equation:

$$y = x^4 + 5x^3 + 5x^2 - 5x - 5$$

This has the right form to be graphed using $x$ and $y$ coordinates. The main point to understand is this:

The solutions to the first equation

$$x^4 + 5x^3 + 5x^2 - 5x - 5 = 0$$

are the $x$ intercepts of the graph of the second equation

$$y = x^4 + 5x^3 + 5x^2 - 5x - 5$$

[1]There are formulas for degree 3 and degree 4, but they are very complicated, and are not very accurate to use on a computer. They are hardly ever used.

So, to solve the first equation, look at the graph of the second and find the $x$ intercepts.

**Step 1.** To see the graph of the second equation, enter the following in the white box at the top of the screen:

```
x^4 + 5*x^3 + 5*x^2 - 5*x - 5
```

Then click on the *Graph Equation* button.

**Step 2.** Observe the location of the $x$ intercepts. How many are there? Zoom in on one of these $x$ intercepts. To do that, right click somewhere on the graph to get a menu, then click on *actions* and then on *zoom in*. Imagine a little box drawn around the point you want to find on the screen. Move the mouse so that the little $\oplus$ is at the upper left-and corner of your imaginary box. Now, press and hold down the left mouse button, and keeping the button down, move the mouse button to the lower right corner of the imaginary box. Then let the button up. The graph will now change to an enlarged picture of what was inside your imaginary box. You may have to click *Graph Equation* again to see the zoomed–in graph.

Now you know how to *zoom in*. You can reverse the process by using the *zoom out* action.

**Step 3.** Using the *actions* menu again, select *get coordinates*. (The little panel that appears can be moved so that it is not covering up what you want to see. Press and hold the left mouse button at the top of the panel and move the mouse to *drag* the panel around the screen.) When you *click* on a point of the graph, the $x$ and $y$ will be shown in the little panel. Click on the $x$ intercept in your figure. The $y$ coordinate should be close to 0. The $x$ coordinate is an approximate solution to the original equation for this activity.

**Step 4.** Repeat the steps above: zoom in a second time on the $x$ intercept on the graph, then get the coordinates of the $x$ intercept. The $y$ coordinate should be closer to 0 than in the previous step, so the $x$ coordinate is a better estimate of the desired solution to the original equation.

**Step 5.** To refine the solution even further, use a numerical method. On the right side of the screen, next to `x =` type in the approximate solution ($x$) to the equation from the previous step. **Warning: do NOT type a carriage return or an enter.** Click on the *Find y* button. Then the $x$ you typed in and the $y$ that goes with that $x$ will be displayed on the screen, and a red point will appear on the graph. If your $y$ comes out to be exactly 0, that means your $x$ is an exact solution to the equation. If $y$ is not exactly 0, you can increase or decrease $x$ to try to make the $y$ closer and closer to 0, and to try to make the red point closer to the $x$ axis.

The window where the $x$ and $y$ values are shown is scrollable. That means you can use the little arrows and buttons attached to the right side of the window to move the text up and down in the window, which is handy if you want to see earlier results. If you use many decimal digits, it will probably be helpful to choose a smaller size type (font) for the white text windows. To do this, right click on the window to get a menu. That will either have the font size directly, or will include a `settings` option that will let you pick the font size. Make the size 10 or less.

Use the button to compute the corresponding $y$. Now change the $x$ to make the $y$ closer to zero. Repeat the process a few times.

**Step 6.** Use these methods to approximate all the solutions to the original equation

$$x^4 + 5x^3 + 5x^2 - 5x - 5 = 0$$

Your answers should be correct to 4 decimal places.

**Step 7.** Write a brief report on this activity. Include in the report the original equation, the new equation with $y$, a graph of the new equation (this can be sketched by hand, or captured with the snapshot feature if you do the report using the computer module), and the approximate solutions you found. For each approximate solution $x$, compute $x^4 + 5x^3 + 5x^2 - 5x - 5$ and show how close the result is to 0.

### Another Equation

Sometimes equations come up in a slightly different form, for example

$$x^4 + 5x^3 + 5x^2 - 5x - 5 = 1.6$$

This is very similar to the previous equation, but instead of 0 in the left side of the equation, the number 1.6 appears. One approach to this situation is to rewrite the equation, using algebra, so that there is a 0 on the right once again. Do that, and then use the same methods as before to find a solution correct to three decimal places. Note that you only need to find one solution this time. Write up your results as before.

### Fitting a Simple Rational Function to Data

In this section of the lab you will use one particular kind of rational function, having the form $y = \frac{a}{x-c} + d$. Here, the $a$, $c$, and $d$ are parameters, and they stand for particular numbers. For example, the following are all possible equations that could be graphed in this part of the lab:

$$y = \frac{2}{x-1} + 5 \qquad y = \frac{1}{x+3} + 4$$

$$y = \frac{3}{x-2} - 1 \qquad y = \frac{4.23}{x-.87} + .029$$

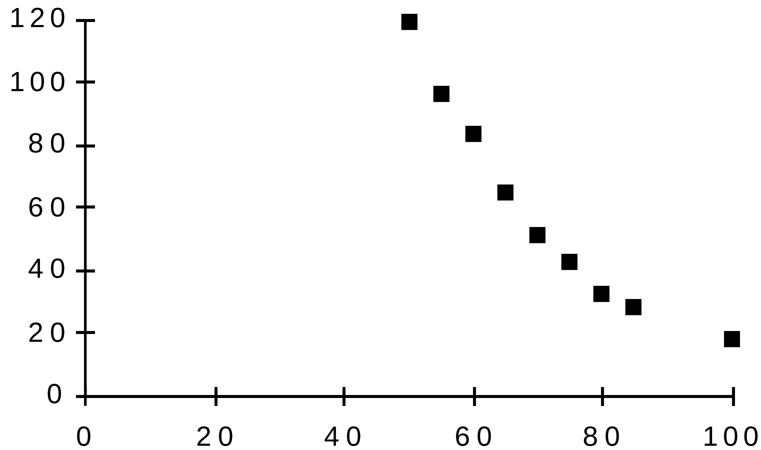
In the first part of this activity, you will learn what happens when you change the values of $c$ and $d$. Then you will use that information to fit a curve to some data points.

**Step 1.** Use the `Window` menu to view the *1overx* module. Follow the directions on the screen. Write a short description in your lab report of what you learn from these activities. When you report on the effects of changing $c$ and $d$, be sure to tell how to predict the locations of the horizontal and vertical asymptotes.

**Step 2.** Use the `Window` menu to view the *graph.mwt* module. Return to the start of the module (use the *menu* button in the lower right corner of the screen), and then click on the *Plot a Data Set* button. The screen that appears will allow you to plot data points as well as the graph of an equation.

**Step 3.** In the table below, some data are shown from a sales survey conducted by Finite Math students in 1994. The $x$ values are prices (in cents) that could be charged for a can of soda, the $y$ values indicate how many sodas could be sold at each price.

| $x$ | $y$ |
|---|---|
| 50 | 119 |
| 55 | 96 |
| 60 | 83 |
| 65 | 64 |
| 70 | 51 |
| 75 | 42 |
| 80 | 32 |
| 85 | 28 |
| 100 | 18 |




Type these data values into the empty table in the upper left–hand corner of the computer screen. You will have to click in the first cell of the table before you type the number 50. After that, you can use the `enter,` `tab,` and arrow keys to get to the other cells of the table.

After you enter the all the data points, you need to change the scales for the $x$ and $y$ axes on the graph. Right click in the graph window to get a menu, and then click on `Settings.` Make the range on the $x$ axis go from 50 to 150. **Note: change the maximum x value to 150** *before* **you change the minimum to 50.** Make the range on the $y$ axis go from 0 to 120. Set the tick mark controls to 10 each. And be sure to change the `function domain` so that it goes from 50 to 150. Click on **OK** to remove the `settings` window.

Next, find and click the button that will show the data points from the table on the graph. You should see something like the graph above. The shape of the data points resembles the graphs you worked with in the preceding exercise, so it is reasonable to look for an equation of the form

$$y = \frac{a}{x - c} + d$$

that comes close to all the data points. For example, enter and graph the equation

$$y = \frac{1500}{x - 30} + 10$$

You will have to type this in the form

```
y = 1500/(x-30) + 10
```

As you know from previous work you can move the graph up and down by changing $d$, and you can move it from side to side by changing $c$. You can also make the graph curve more or less by changing the value of $a$. To see that, change the equation you just graphed to

$$y = \frac{2500}{x - 30} + 10$$

and then look at that graph. By choosing different values for $a$, $c$, and $d$, increase or decrease the amount of curve, and move it up, down, left, or right, until it fits closely to all the data points.

After you find a curve that seems to come close to the data points, write a short report giving your results. If you use the computer report module, include a copy of the graph with the data points and your curve. Otherwise, sketch your curve as accurately as possible on the graph above on this sheet and put a copy in your report. Your report should also give the equation for your best fitting curve.

**Graphical Properties of Rational Functions**

In this section, if there is enough time, you will use the graphing module to explore graphs of rational functions.

**Step 1.** Return to the *2D graphs* screen in the graphing module. Use that to examine each of the following graphs:

$$y = 1 + \frac{2x - 1}{x^2 - 2x - 3} \qquad y = \frac{x^2 - 4}{x^2 - 2x - 3} \qquad y = \frac{(x-2)(x+2)}{(x-3)(x+1)}$$

Remember that on the computer a fraction has to be typed using the form `()/()` and that you must use * to indicate multiplication. For example, the first equation would be typed like this

```
y = 1 + (2*x - 1)/(x^2 - 2*x - 3)
```

What do you observe about the graphs of the three equations?

**Comment** All three graphs for the equations above are identical, because while the equations look quite different, they are algebraically the same. That means, for any numerical value of $x$, the corresponding value produced for $y$ from each equation will be the same result. Now we will consider what the different forms tell us about the graph.

**Step 2.** What are the $x$ intercepts of the graph? Which of the three equations can be used to find the $x$ intercepts most easily? How?

**Step 3.** What is the $y$ intercept? Which equation can be used to find this most easily? How?

**Step 4.** Where are there vertical asymptotes? Which equation can be used to find this information most easily? How?

**Step 5.** Where is there a horizontal asymptote? Which equation can be used to find this information most easily? How?

**Step 6.** Write into the lab report a brief description of what can be learned from different forms of an equation, and use the equations from this exercise as an example to illustrate your remarks.