Elementay Math Models

Lab Activity for Chapter 9

This laboratory period will be spent working on a few problems in context exercises from Chapter 9. Work on each problem as if you were doing it for homework. Use the computer to obtain graphs and numerical data values, and to find solutions to equations using graphical and numerical methods. Then, either copy the computer results onto your homework paper by hand, or make a snapshot, put it into a report page or a word processing document, and print it out.

Problems in Context 6

1. Use the diffequ.mwt module to see what happens for the model in this problem. Remember that the difference equation $a_{n+1} = .5a_n$ gets entered in the computer module in the form

$$\mathtt{a(n+1) = .5 * a(n)}$$

2. The problem in the book asks how much aspirin will be in the blood after 4 hours. You can use the model to answer this. What should you use for n? Be careful, the answer is **not** 4. Reread the description of what a_n represents in the model. Using similar methods, how much aspiring is in the blood after 2 hours? After 6 hours? After 8 hours? Make a table on your homework paper showing these results.

3. Based on your results, does proportional reasoning apply to this model? Your results should show that $a_0 = 750$ and $a_4 = 46.875$. Using proportional reasoning, what would you expect the value of a_2 to be? Compare that to the value in the table for a_2.

4. Find an equation that gives the amount of aspirin in the blood after t hours. Use the boxed statement on page 195 of the text. For this problem, what is a_0? What is r? What is d? Remember that t is supposed to be in units of hours.

5. Use your equation to compute the amount of aspirin in the blood after 4 hours ($t = 4$.) After 6 hours. After 8 hours. Compare the results to question 2 above.

6. Your answer to question 3 should be $a(t) = 750(.5)^{t/.5}$. Use the graph.mwt module to examine the graph of this equation. You will have to use x and y for the variables in the equation: change the t to an x and replace $a(t)$ with y. Remember that you must use an asterisk: $750 * (.5)^{t/.5}$

7. If the patient takes 500 mg of aspirin instead of 750, how would you expect that to affect the answers to the problems above? How does the equation change for $a(t)$? Predict how the graph should change. Then enter the new equation in the graph.mwt module and compare the graph to the original graph.

8. In the original model, it is stated that half of the aspirin in the blood is removed every half hour. Suppose that assumption is changed. Suppose we find that 3/4 of the aspirin in the blood is removed every half hour. How would you expect that to affect the graph of $a(t)$? How does the equation for $a(t)$ change? Enter this new equation for $a(t)$ and look at the graph. Were your predictions correct?

Problems in Context 7

1. For this problem, you have to begin by setting up the model. Given the information in the problem, it is natural to set the model up with one data value every four hours. If an athlete

takes an initial dose of 16mg, and if the amount of steroids in the blood is measured every 4 hours, what would be the amount for the first measurement? For the second? For the third? Use this information to create a difference equation model. Let a_n stand for the amount of steroids in the blood after n four hour periods have gone by. What is the difference equation for a_n? What is a_0?

2. Use the diffequ.mwt module to explore your model. Remember that you will have to change your difference equation to the proper form:

$$\mathtt{a(n+1)} = ? * \mathtt{a(n)}$$

   Looking at your graph and data, when does a_n first fall below 1 mg? How many hours is that after the drug was taken? When does a_n first fall below .01 mg? How many hours is that after the drug was first taken?

3. Find a functional equation for a_n. Graph it using the graph functional equation button. If your functional equation is correct, it should exactly match the original graph for your model.

4. Find an equation for your model using the form shown in the box on page 195. Let $a(t)$ be the amount of drug in the blood t hours after the drug was taken. Graph this new equation using the graph.mwt module.

5. Use the graph and numerical methods to figure out how many hours it takes for an initial does of 16mg to be reduced to .01 mg. What does this imply about trying to use a blood test to determine if an athlete has taken drugs?

Problems in Context 8

1. Use the diffequ.mwt module to explore the difference equation given in this problem. Remember to change it to the form

$$\mathtt{a(n+1)} = \mathtt{.75} * \mathtt{a(n)} + \mathtt{100}$$

   Is this a geometric growth model? Why or why not? You should see that the graph levels off. How does this relate to the data in the table? Based on this, what would you predict as the long term effects of taking 100mg of the drug every four hours?

2. Suppose the patient takes 200 mg every four hours instead of 100mg. How do you think that would change the model? How does the difference equation change? How does the graph change? How does the data table change?

3. Explore several different versions of this model using different amounts for the dose taken each four hours. Make a table showing the different doses you consider, and for each dose, the amount of drug in the blood where the model levels off. Do you see a pattern? Can you predict what dosage should be given if the doctor wants the amount of drug in the patient's blood to level off at 150mg?

4. **[Extra Credit]**. In the model so far, it is assumed that 1/4 (or 25%) of the drug is removed by the body every four hours. In this problem, explore what happens if that assumption is modified. What if we assume that 20% is removed every 4 hours? What if we assume that 30% is removed? Make a table showing how the leveling off point depends on both the percentage removed every four hours and the size of the repeated medicine dose. Can you develop a formula that predicts the level off point given the percentage removed every four hours and the dose size?