



Sum Formulas for Sine and Cosine

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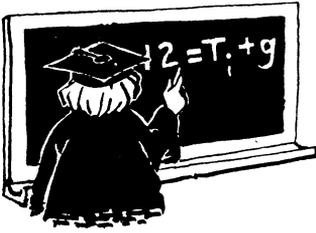
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CLASSROOM CAPSULES

Edited by
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Sum Formulas for Sine and Cosine

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One of the treasures of antiquity is a diagram which quickly yields a proof of the Pythagorean Theorem. In a similar spirit, Figure 1 can be used to prove some other important results in trigonometry. It remains only to indicate proper labeling. Since the diagonal of the inscribed rectangle is 1, its sides measure $\cos \beta$ and $\sin \beta$. These constitute hypotenuses for the similar triangles in the four corners of the large rectangle. Therefore, looking at triangle ABC in Figure 2, we have:

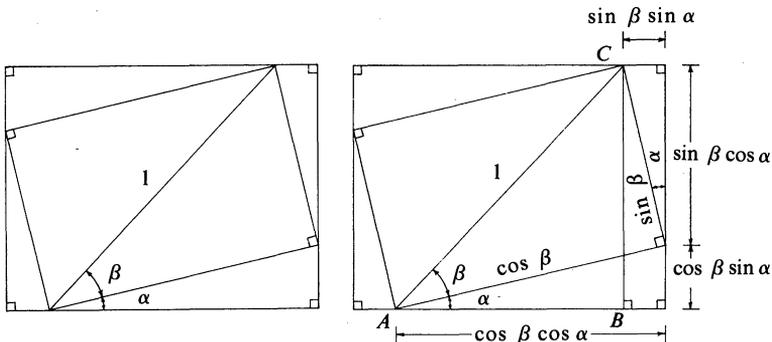


Figure 1.

Figure 2.

$$\sin(\alpha + \beta) = \cos \beta \sin \alpha + \sin \beta \cos \alpha$$

and

$$\cos(\alpha + \beta) = \cos \beta \cos \alpha - \sin \beta \sin \alpha.$$

Figure 2 is valid for acute α and β with sum less than 90° , the only domain that is meaningful for the trigonometry of right triangles. However, adopting the usual interpretation of reference angles, we can modify Figure 2 to establish the addition formulas in general for acute β . Clearly, the inscribed rectangle may be constructed for any acute β . Hold the lower left-hand corner of this rectangle at the center A of the unit circle and rotate through any angle α so that the bottom of the rectangle becomes the terminal side. Then construct the circumscribing rectangle with sides parallel to the coordinate axes. Figure 3 illustrates one such situation with angle $\alpha + \beta$ in the third quadrant. As before, the dimensions of the small triangles in the corners are simply related to α' , the reference angle for α . Thus, these dimensions may be specified in terms of α as in Figure 3. Now the coordinates of C are determined, giving the sine and cosine of $\alpha + \beta$. For other orientations of the inscribed rectangle, similar arguments may be used.

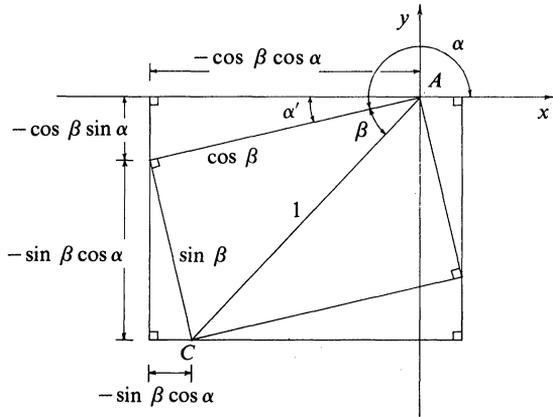


Figure 3.

As a special case, take β to be the complement of α . Then Figure 2 becomes Figure 4. Thus, we obtain the identity $\sin^2\alpha + \cos^2\alpha = 1$, and a proof of the Pythagorean theorem follows.

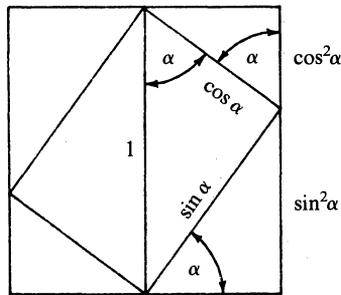


Figure 4.

