

Preface

This book is a travel guide to three mathematical realms. It is not for beginners. The intended audience includes those who are already very familiar with the most popular attractions of the three realms. So why do they need a travel guide at all?

By way of analogy, think about tourists who travel to France. Many will visit France just once, or perhaps a few times. On these visits they will take in the most popular and famous sights and destinations: the Eiffel Tower, the Louvre, and so forth. But there are others who visit France many times, forging a special fondness for the place and its people. These travelers have been to all the most popular attractions, and now wish to expand their French experience by seeking out lesser known destinations.

It is much the same with mathematics. Most who study beyond the minimum demanded by high school or college follow a common path through a standard curriculum: algebra, precalculus, calculus, and perhaps as far as linear algebra and differential equations. These students are like those who visit France just once or twice. In their limited exposure to the subject, they focus on the main attractions. This book is not for them, or at least, not primarily for them.

Rather, it is for their teachers. The teachers have traveled through the standard curriculum repeatedly, until it is thoroughly comfortable and familiar. When it comes to touring these mathematical realms, they are old hands, indeed. And like those many-time visitors to France, they may wish to seek out new vistas and explore unsuspected wonders a bit off the beaten track.

It is also for anyone who has a long history of applying mathematics, including scientists, engineers, and analysts. They, too, are intimately acquainted with the core ideas of algebra, calculus, and related subjects. They, too, might be interested in seeing some new aspects of an old familiar terrain.

The mathematical subjects that contain the standard curriculum are incredibly rich. There are countless extensions, digressions, and supplementary topics that illuminate and enrich the core. Moreover, these topics are highly accessible to the seasoned mathematical hands who have journeyed so many times through these realms. Anyone who has taught precalculus a few times will be well prepared to explore additional properties of polynomials, rational functions, and exponentials, for example.

In fact, there are so many possible journeys into the mathematical back country, as it were, that it would be impossible to survey even a small percentage of them in a single volume. This book makes no pretense of doing so. Rather, I have gathered together some topics that are particularly appealing and familiar to me, in the hope that these topics will be interesting and appealing to others, as well.

The title and organization of the book follow the metaphor introduced above. The material is presented as a series of explorations in three mathematical realms, focussing on polynomials, max/min problems, and calculus. A thorough familiarity with the standard curriculum should be sufficient background for all of the excursions. However, for those who wish to brush up on parts of this material, or who might be accustomed to different notations or terminology than I use in the main text, I have prepared some appendices. These are available at a companion website for this book, see [87]. There the reader will also find some additional supplementary material, including animated and interactive graphics displays for some of the topics discussed in the book.

Although this book was written with teachers in mind, it is really for anyone who holds a deep affection for the core curriculum, and wishes to study more of it. This can include specialists in allied fields who retain a strong mathematical inclination, as well as students who are particularly attracted to the subject. They will find curious connections, surprising patterns, and unexpected insights about old and familiar ideas.

So pack your bags, get a stout walking stick, and meet me at the portal on page 1. Wonders await!