Exercises 1

For any polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

1. Prove the sum of the roots is $-a_{n\text{-}1} / a_n$

2. Prove the average of the roots of p is equal to the average of the roots of the derivative p'.

Exercises 2

1. Prove this: For any polynomial p of degree n with nonzero constant term,

   Rev $p(x) = x^n\, p(1/x)$.

2. Use 1 to prove that the roots of Rev $p(x)$ are the reciprocals of the roots of $p(x)$.

3. Use 2 to prove: if

   $p(x) = a_n x^n + \cdot\cdot\cdot + a_1 x + a_0$

   and $a_0 \neq 0$, then the sum of the reciprocals of the roots of p is $-a_1 / a_0$

Exercise 3

Prove that

$$\frac{\text{Rev } p'(x)}{\text{Rev } p(x)} = s_0 + s_1 x + s_2 x^2 + s_3 x^3 + \cdots$$

where s_k is the sum of the k^{th} powers of the roots of p.

Hints: Use (and prove as necessary) the following:

- Rev $f(x) = x^n f(1/x)$ when f is a polynomial of degree n and $f(0) \neq 0$.
- Logarithmic Derivative: $f' / f = (\ln f)'$
- If $f(x) = (x - r)(x - s)(x - t) \cdots$ then $(\ln f(x))' = \frac{1}{x-r} + \frac{1}{x-s} + \frac{1}{x-t} + \cdots$
- Geometric Series: $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots$ and $\frac{1}{x-a} = \frac{-1/a}{1 - x/a}$
- But caution: Rev $f'(x) \neq (\text{Rev } f(x))'$