Virtual Mathematics

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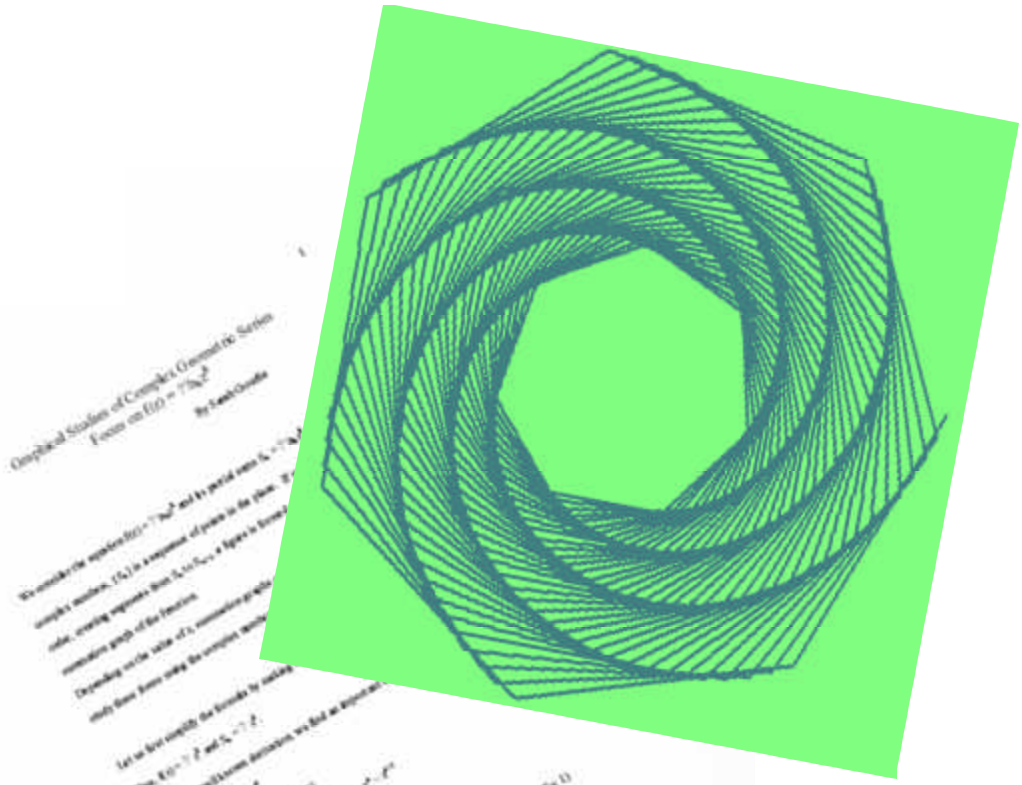
OUTLINE

- Overview
- For Students
- For Us
- In the classroom
- It’s Easy!

Overview

- Visualization
- Graphics, Animation, Sound, Color, Symbolics, Numerics
- Virtual Reality for Mathematics – Virtual Ideality
- Mathwright

Sarah's Summer Project



Convergence Graphs

- Inspiration: Applet for $\sum z^k$
- Goals: Explain and predict patterns for this and other series
- Method: create Mathwright page to visualize convergence patterns for specific values of z
- Sarah picked up Mathwright quickly

Basic Shapes

z = 1e^(m/n)2pi

Length is kept constant at 1.
Angle is a rational fraction of 2pi.

Change the angle by choosing m
and n for a rational fraction in

m 18

n 23

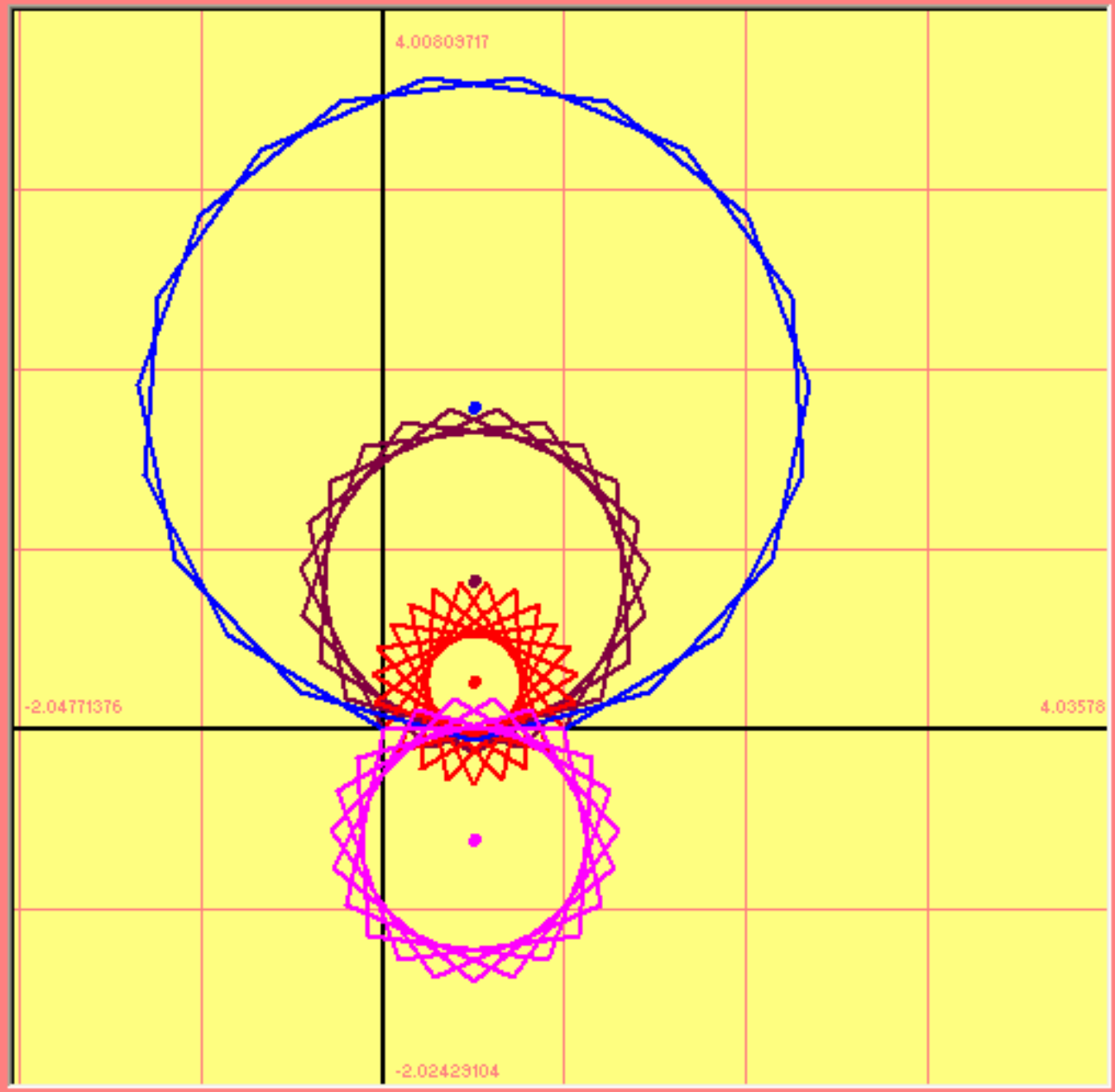
draw | clear | center point

CENTER:

how coordinate

circumscriber

next



Spiral

back

z = re^i(1/n)2pi + .01

Spirals are formed by the basic shapes. The shapes rotate and get a little bigger every time.

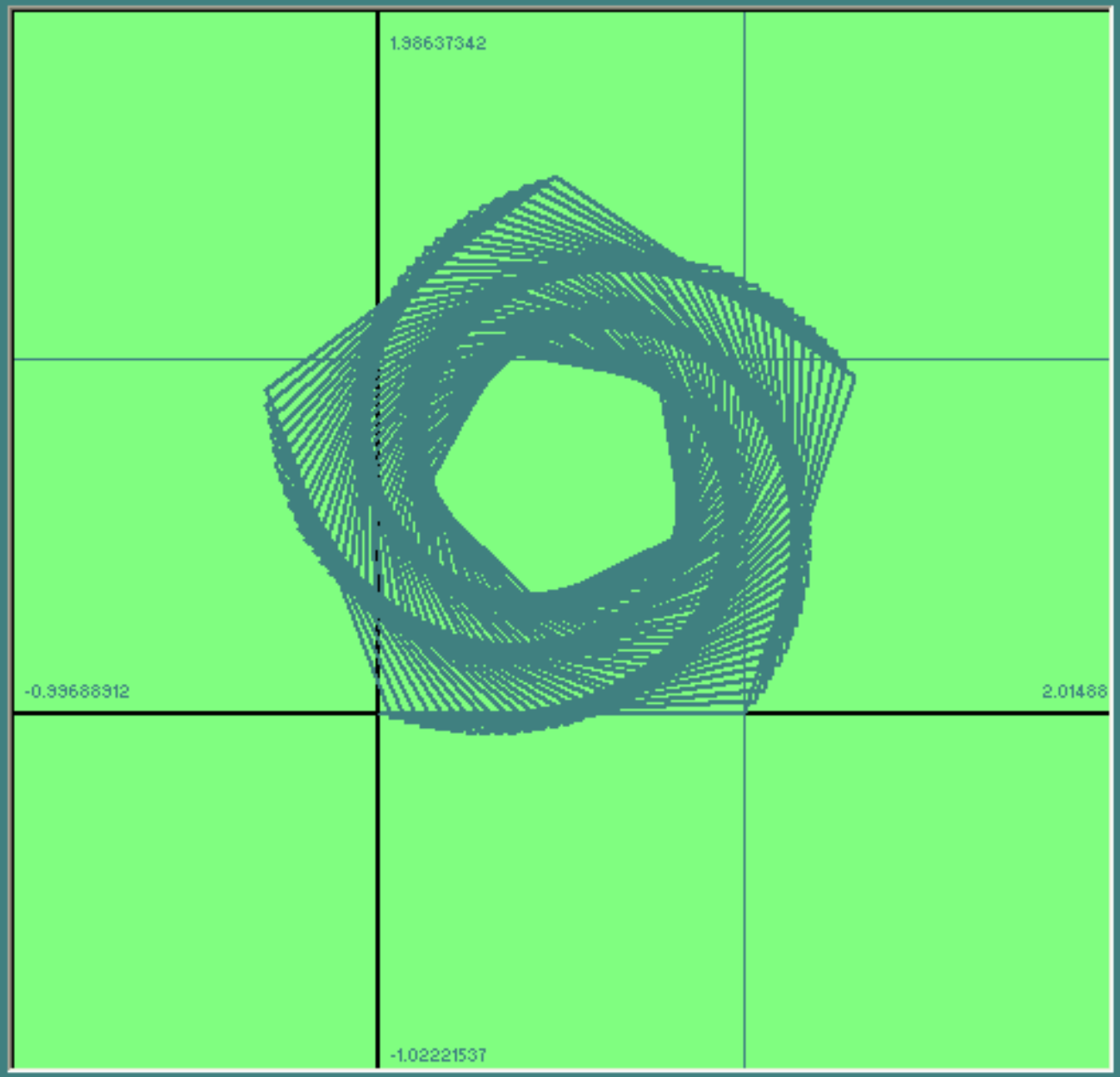
Specify how many arms in the spiral and then adjust

number of arms 5

adjust the length

9973.0

draw clear



Spiral

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Spirals are formed by the basic shapes. The shapes rotate and get a little bigger every time.

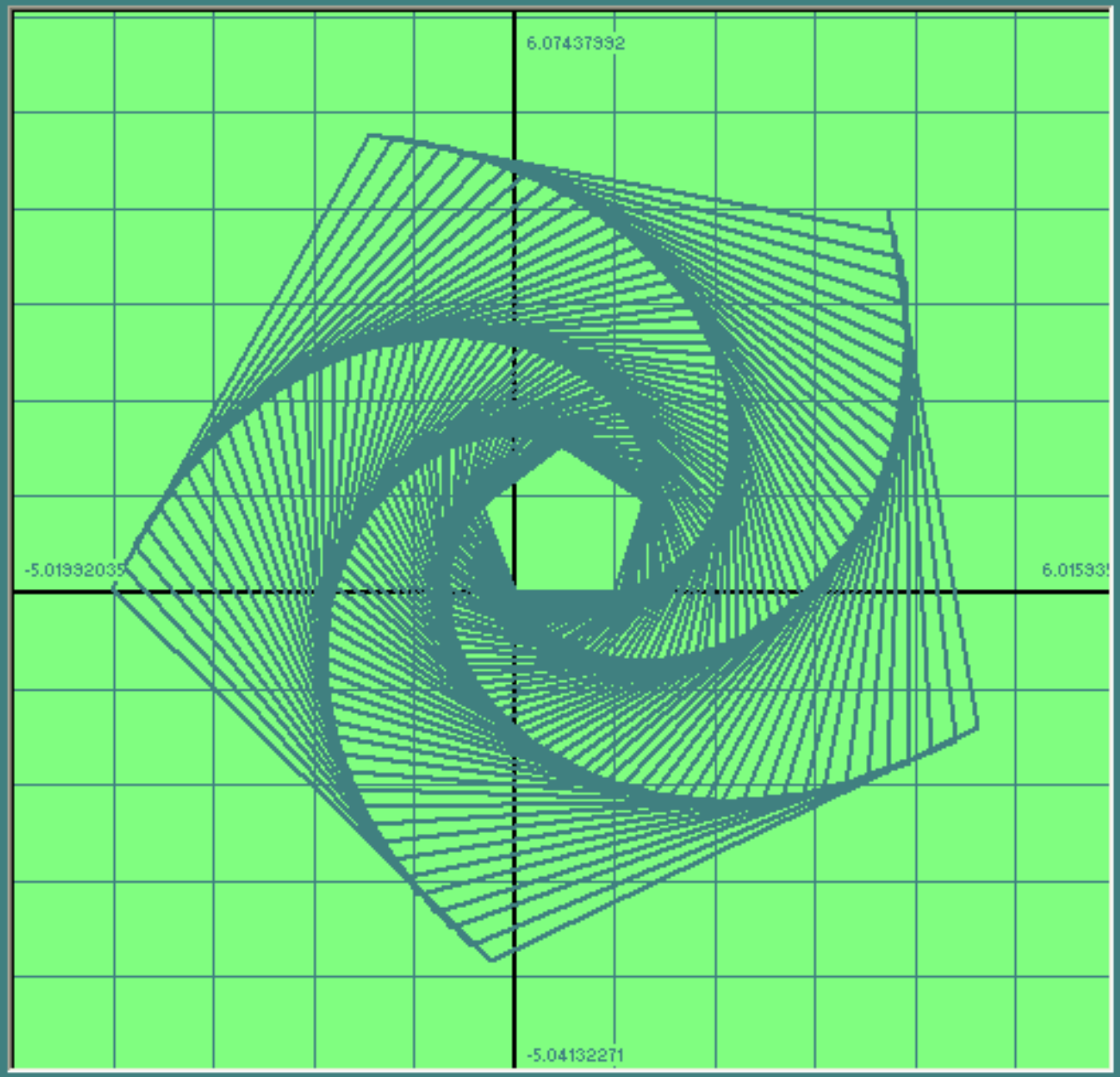
Specify how many arms in the spiral and then adjust

number of arms 5

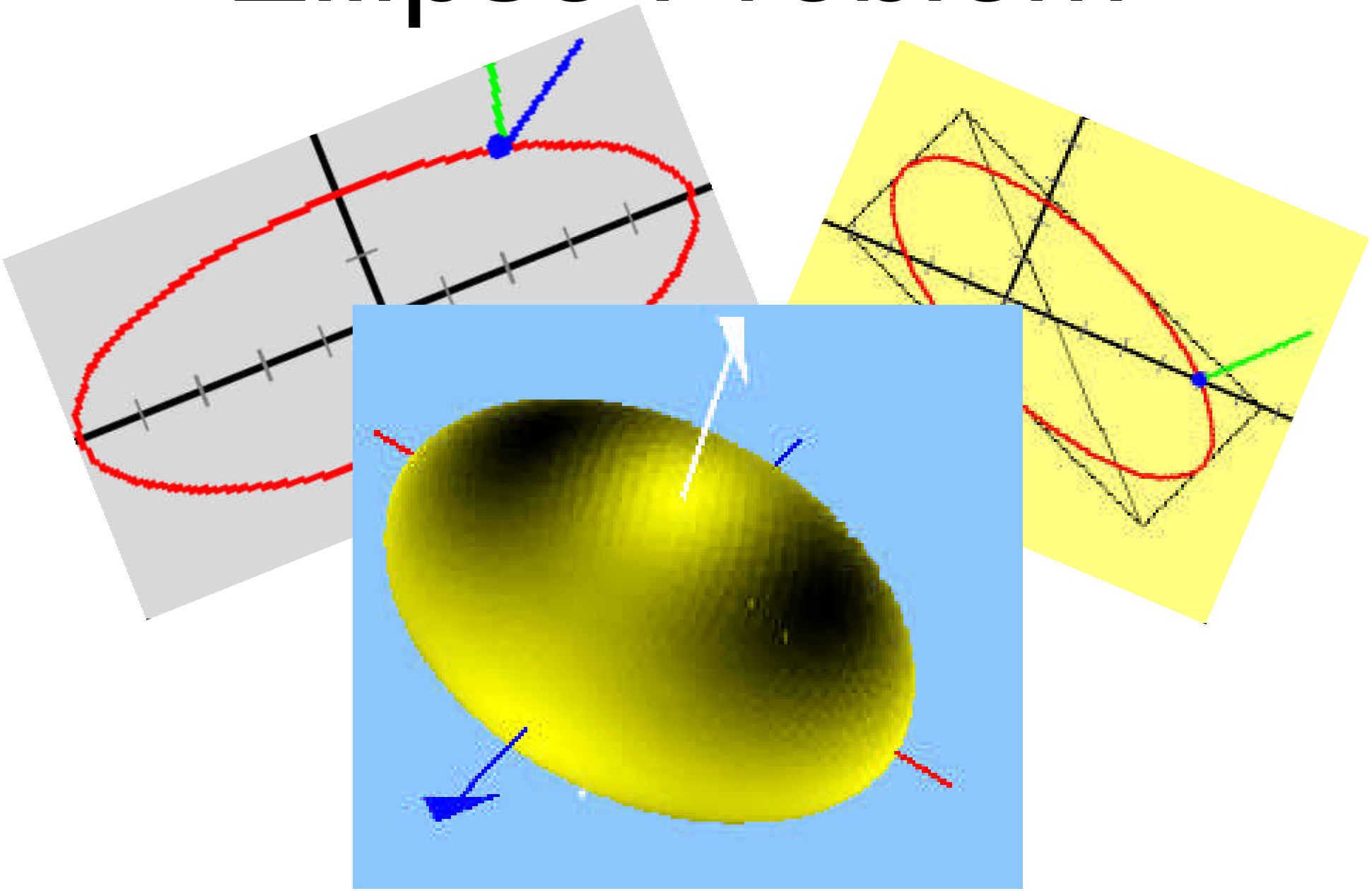
adjust the length

10057.0

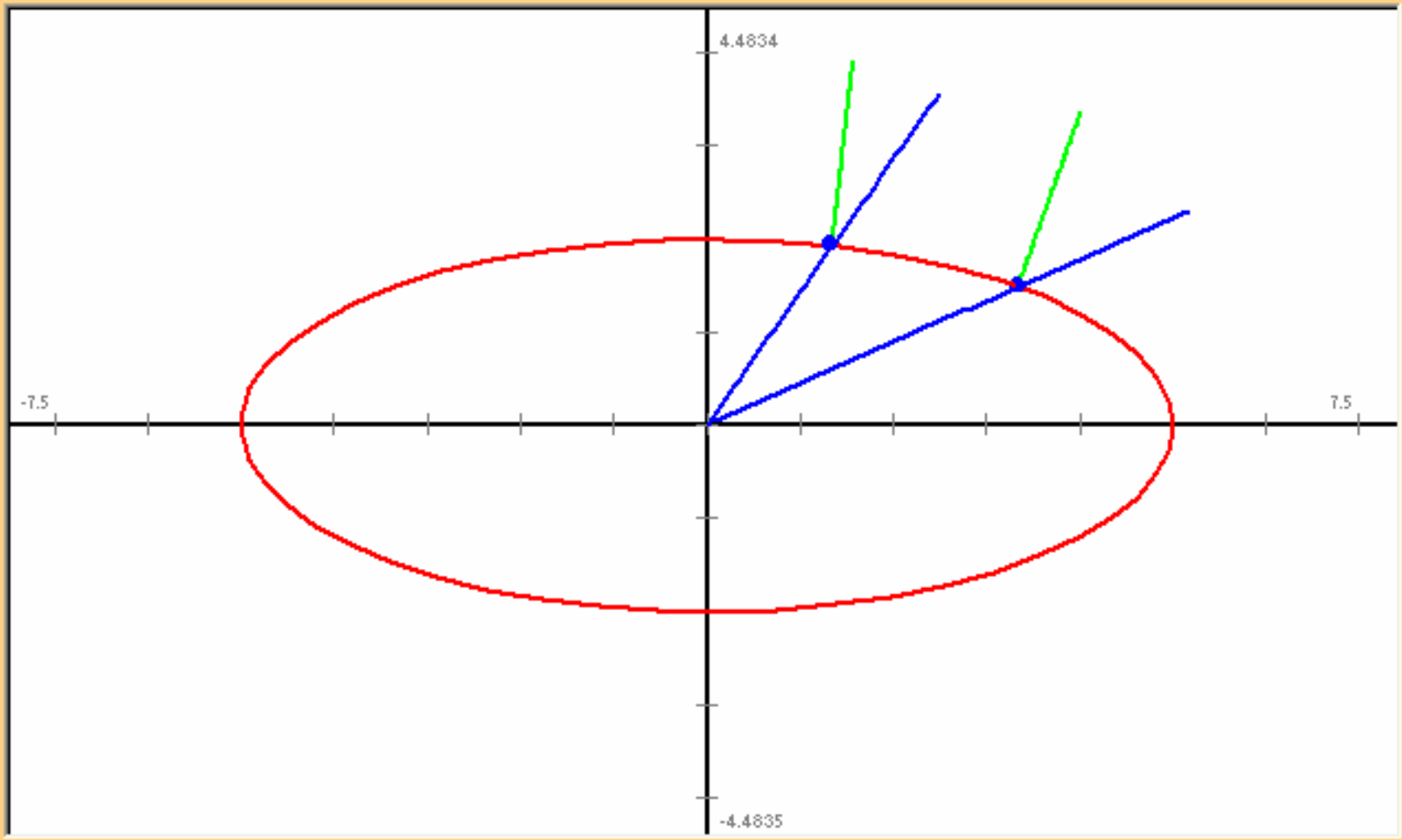
draw clear



Ellipse Problem



- Problem Statement: Find max angle between normal and radial vector on an ellipse
- (Show Mathwright Here)
- Nice geometric construction of solution
- Mathwright visualizations
- Extension to n dimensions



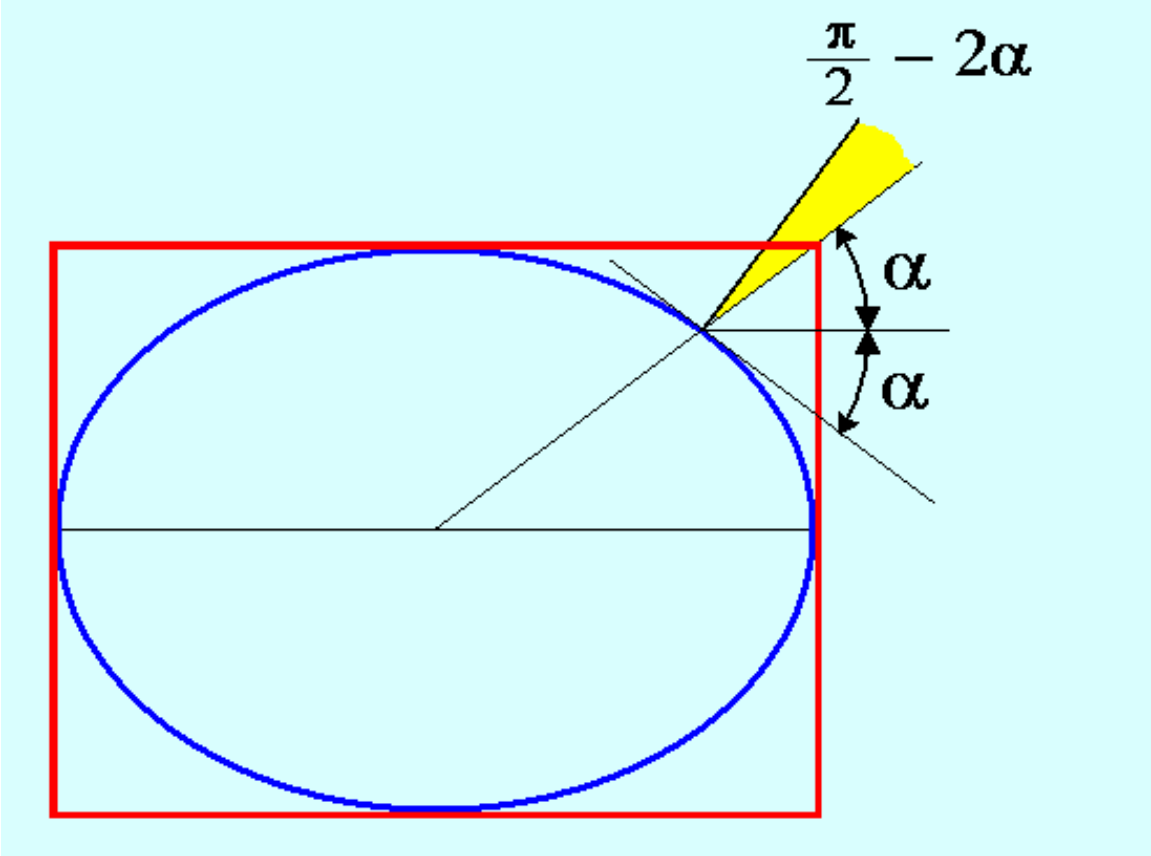
Draw Ellipse

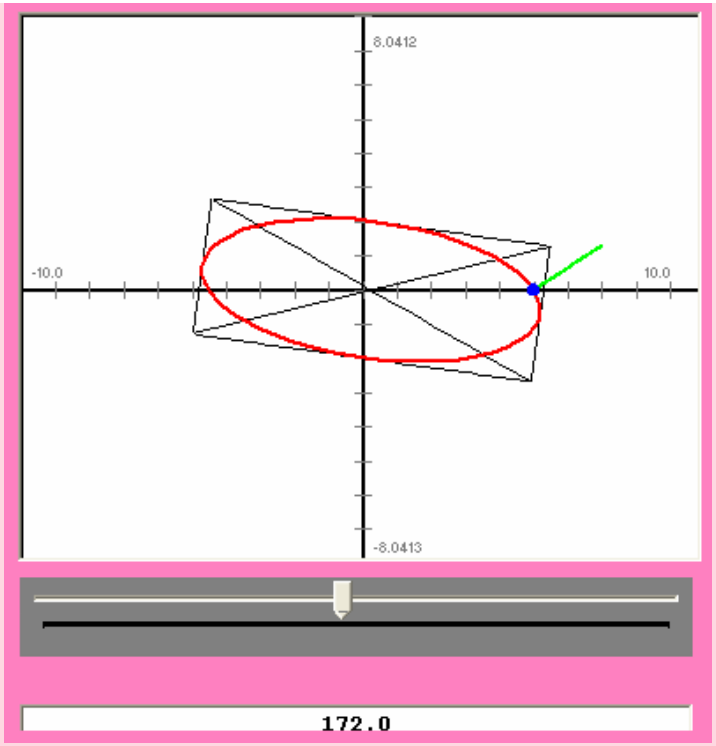
Clear

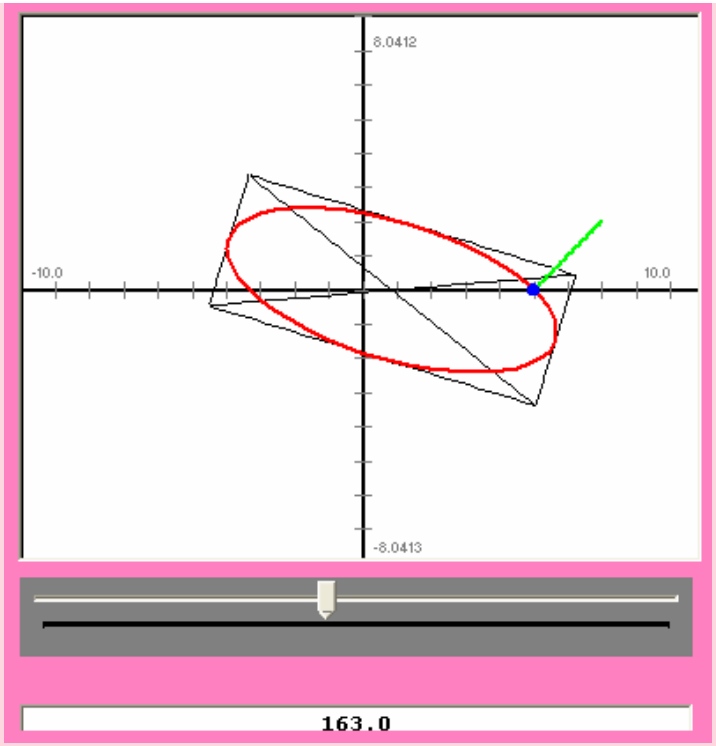
Next

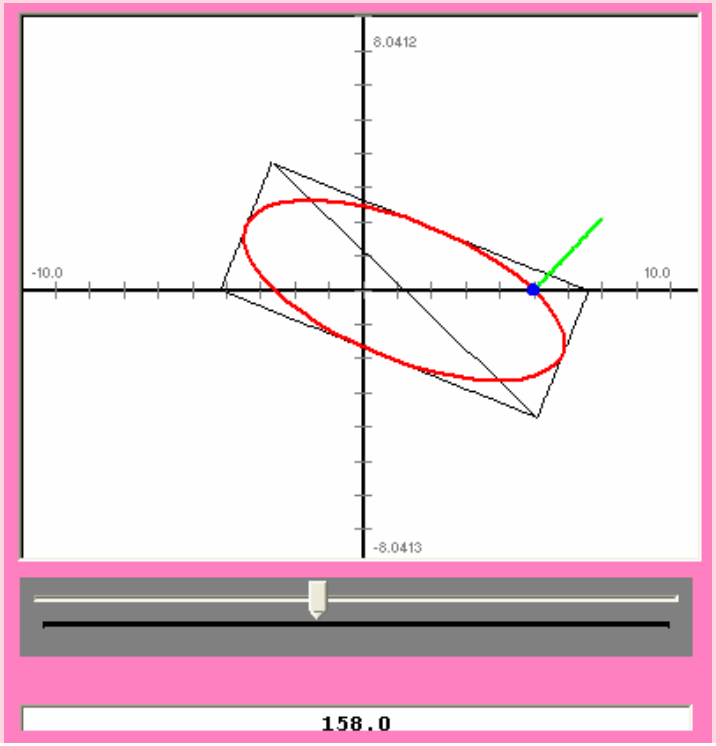
24.0

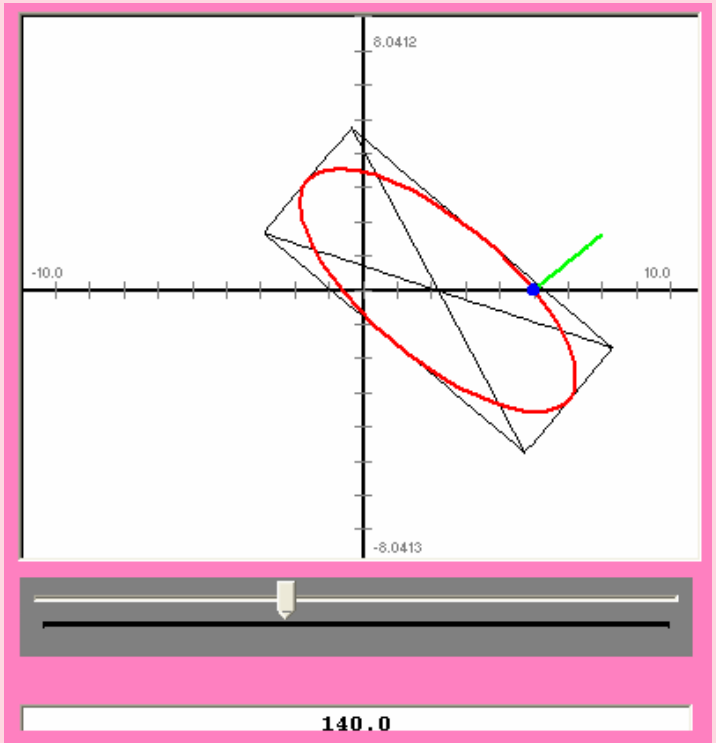
Geometric Construction




8.0412
-10.0
10.0
-8.0413
172.0


8.0412
-10.0
10.0
-8.0413
163.0


8.0412
-10.0
10.0
-8.0413
158.0


8.0412
-10.0
10.0
-8.0413
140.0

3 Dimensional Case

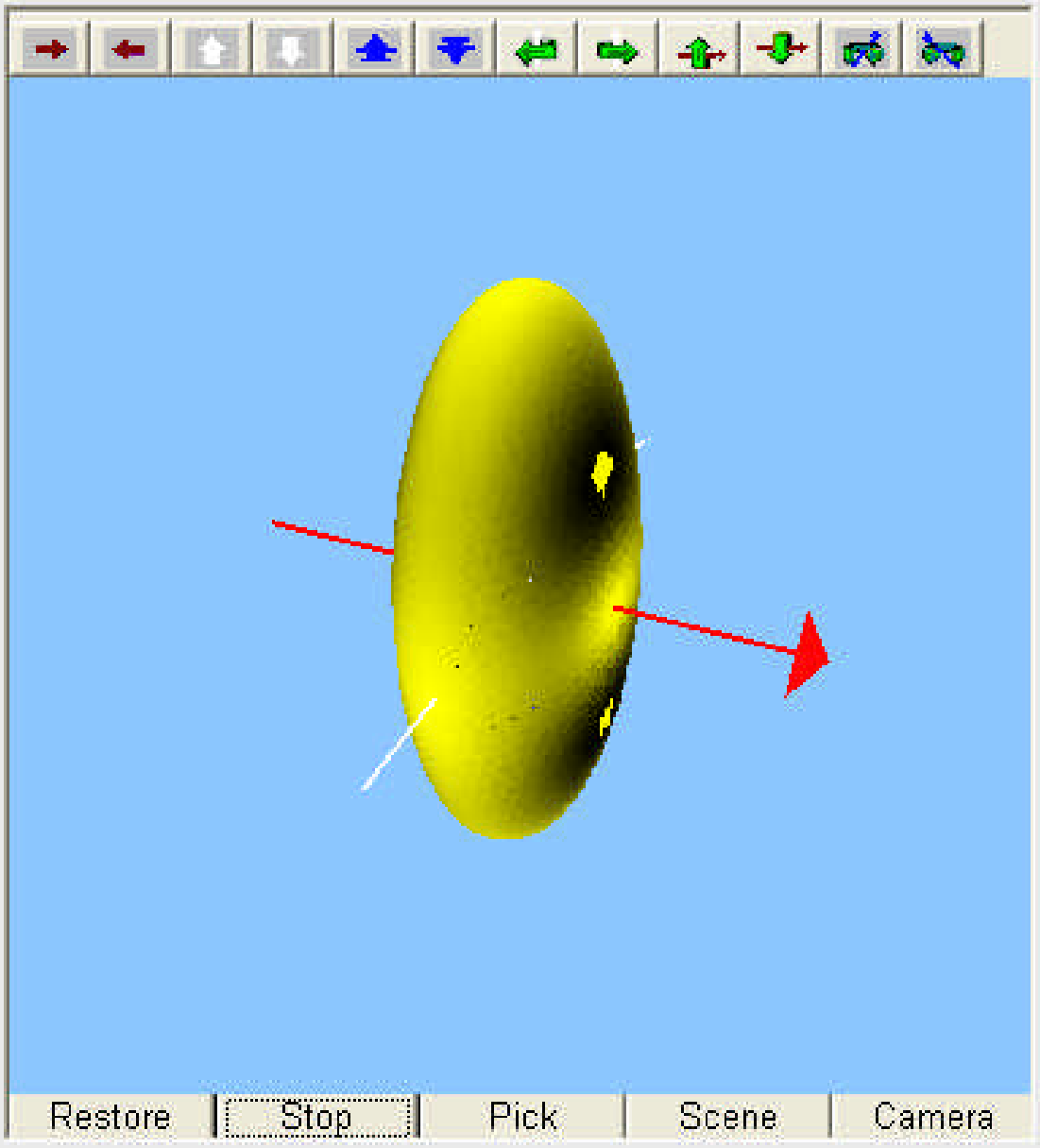
- How’s your intuition?
- There is a simple extension to n dimensions, but …
- It isn’t the corner of the superscribing box
- Visualization helps
- (Show 3D Mathwright Page)

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

draw

calculating intensities...

minimum cosine is 0.8456360
rendering...



In the classroom

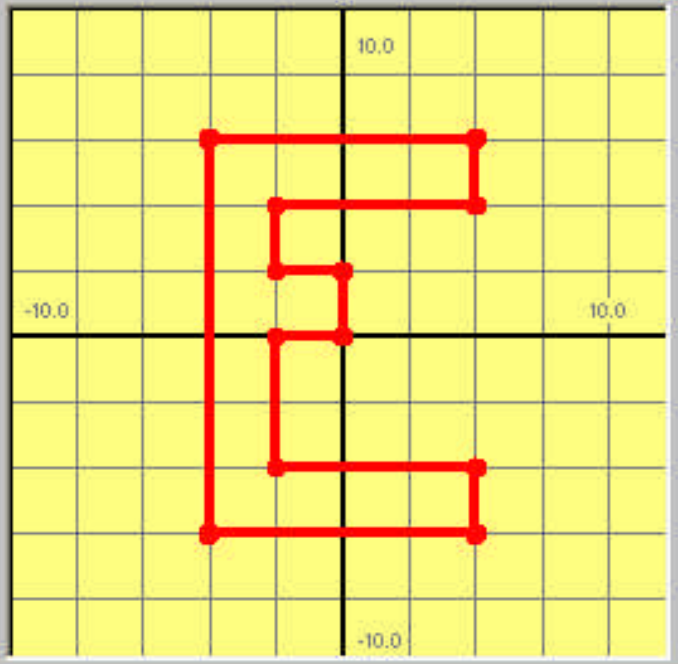
- Empirical Investigation –
  Virtually
- User Interface Significance
- Students can discover relationships and principles by seeing them in action
- Carefully chosen examples can also reveal the limitations of empiricism and the need for theory

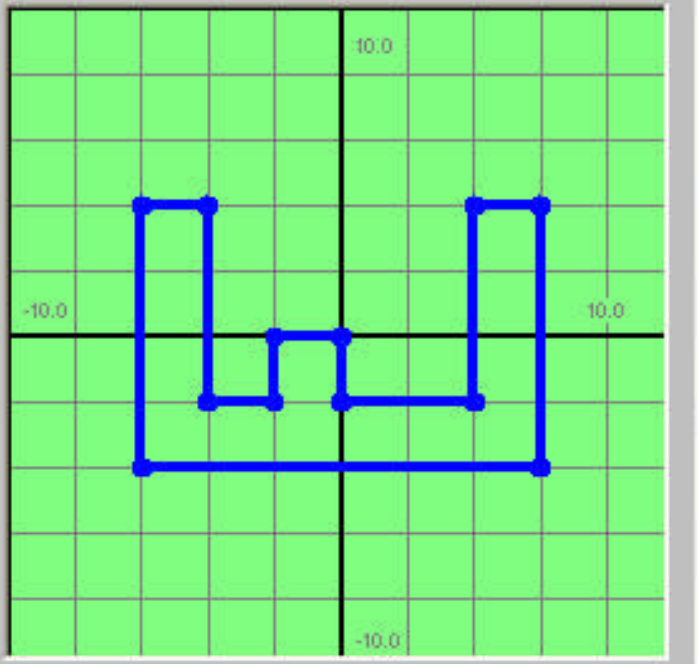
$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

click points | polygon | circle | drag point | Change A | clear

☑ Show Position Vectors For Drag Operation ☐ Restrict to Vectors of Length 4

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} * \begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$$
$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} * \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$
$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} * \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} * \begin{bmatrix} -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$
$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} * \begin{bmatrix} -2 \\ -4 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$
$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} * \begin{bmatrix} 4 \\ -4 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$
$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} * \begin{bmatrix} 4 \\ -6 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$
$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} * \begin{bmatrix} -4 \\ -6 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \end{bmatrix}$$





polygon Click on points in the yellow graph to define a polygon. For each point v of the polygon, A*v will be shown in the green graph, and the numerical results will be printed at left.

Double-Click The Left Mouse Button in the Yellow Window To Stop

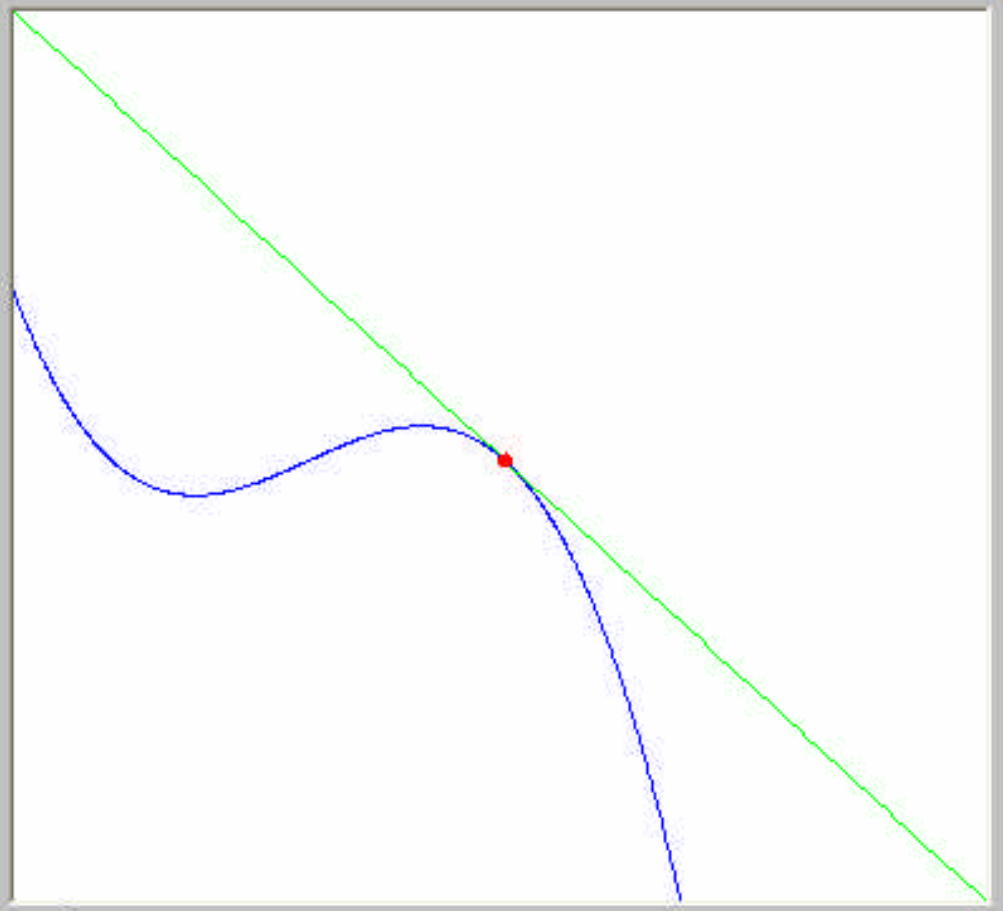
Next Page

f(x) = .5*x - x^3/8

-1.0

Slope of Green Line

max y =
5.000000



min x =
-3.000000

max x =
7.000000

min y =
-5.000000

zoom in at x = 2

magnify 2

Show Instructions

Restart | Zoom In

Clear | Zoom Out

Limits | Overview

It’s Easy!

- Point and click composition idiom
- Simple scripting language
- (Show a demo here)
- Exports to webpages
- Activities are *Open Source* so collaboration (and borrowing) are easy

Mathwright

- Created by James White
- Featured software for MAA IMTP, Internet Library Project, Project WELCOME, MATHDL
- Website: www.mathwright.com
- Only for MS Windows

James White

1946 - 2004